Design of Experiments

The Bridge to Systematic Innovation

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2010 ITEA Annual Symposium
Glendale, AZ
13 September 2010

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Introductions

- **Name**
- **Organization**
- **Job Title/Responsibilities**
- **Experience in T&E, Combinatorial Testing, DOE, etc.**

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Agenda

- Some Basic Definitions and Terms
- Various Approaches to Testing Multiple Factors
- Design of Experiments (DOE): a Modern Approach to Combinatorial Testing
- Examples and Demonstration of a DOE
- Using DOE to Achieve Design Optimization
- DOE with Computer Simulation
- Testing a Very Large Number of Factors
- High Throughput Testing

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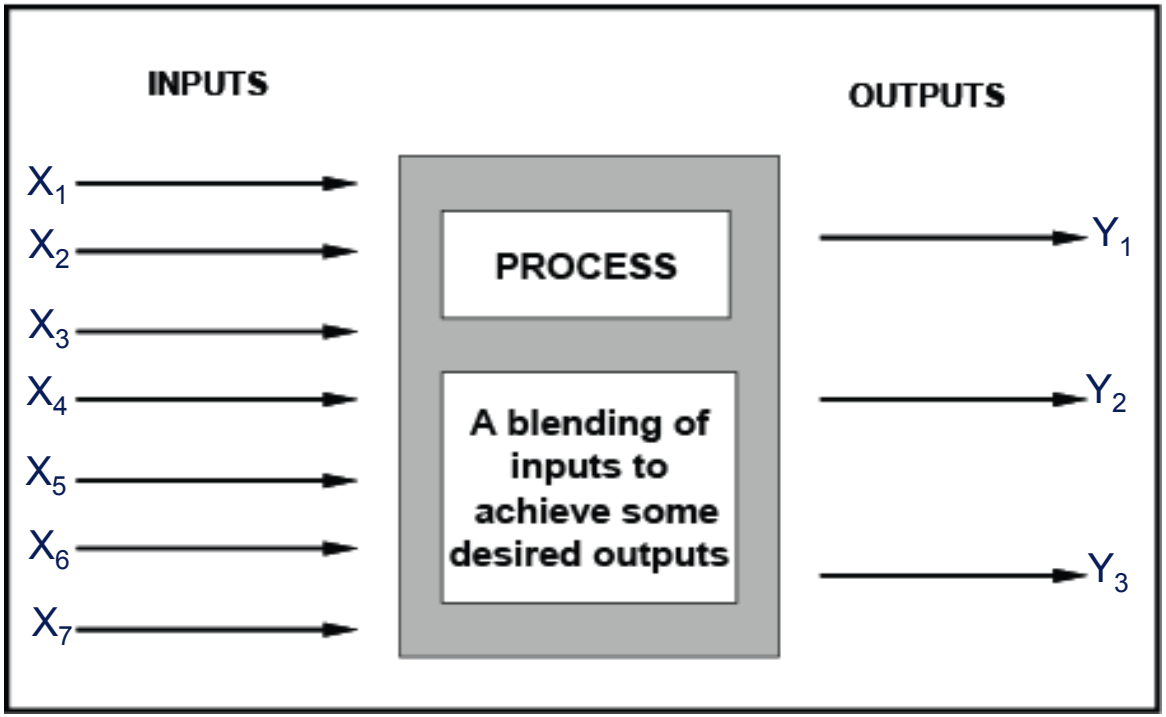
Definition of a Process



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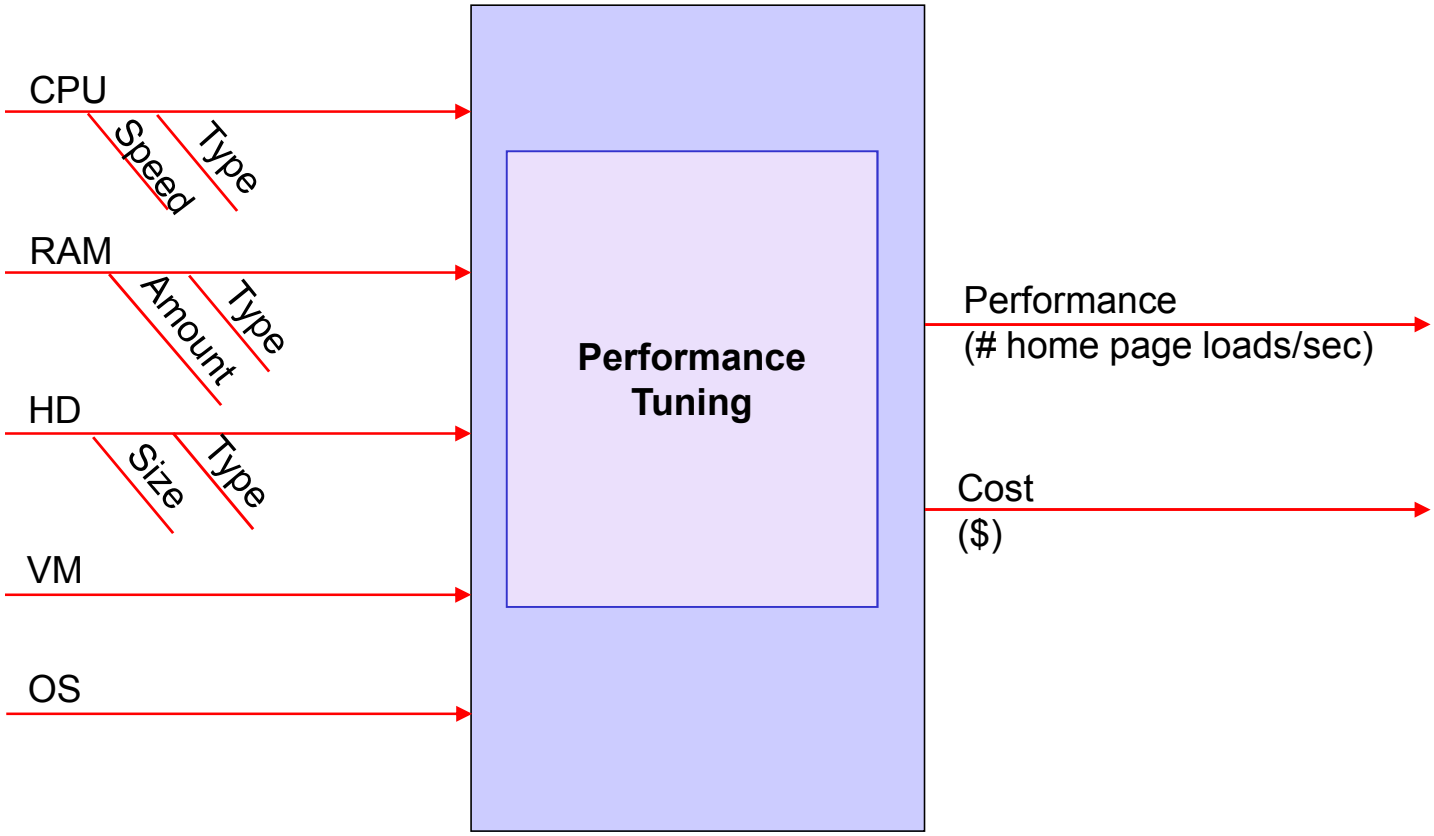
Web-Based Application Process



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Combinatorial Test Terminology

Y: Output, response variable, dependent variable

X: Input, factor, independent variable (a measurable entity that is purposely changed during an experiment)

Level: A unique value or choice of a factor (X)

Run: An experimental combination of the levels of the X's

Replication: Doing or repeating an experimental combination

Effect: The difference or impact on Y when changing X

Interaction: When the effect of one factor depends on the level of another factor

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Performance Tuning Terminology

Factors/Inputs (X's)	Levels (Choices)	Performance/Outputs (Y's)
CPU Type	Itanium, Xeon	# home page loads/sec
CPU Speed	1 GHz, 2.5 GHz	Cost
RAM Amount	256 MB, 1.5 GB	
HD Size	50 GB, 500 GB	
VM	J2EE, .NET	
OS	Windows, Linux	

Which factors are important? Which are not?
Which combination of factor choices will maximize performance?
How do you know for sure? Show me the data.

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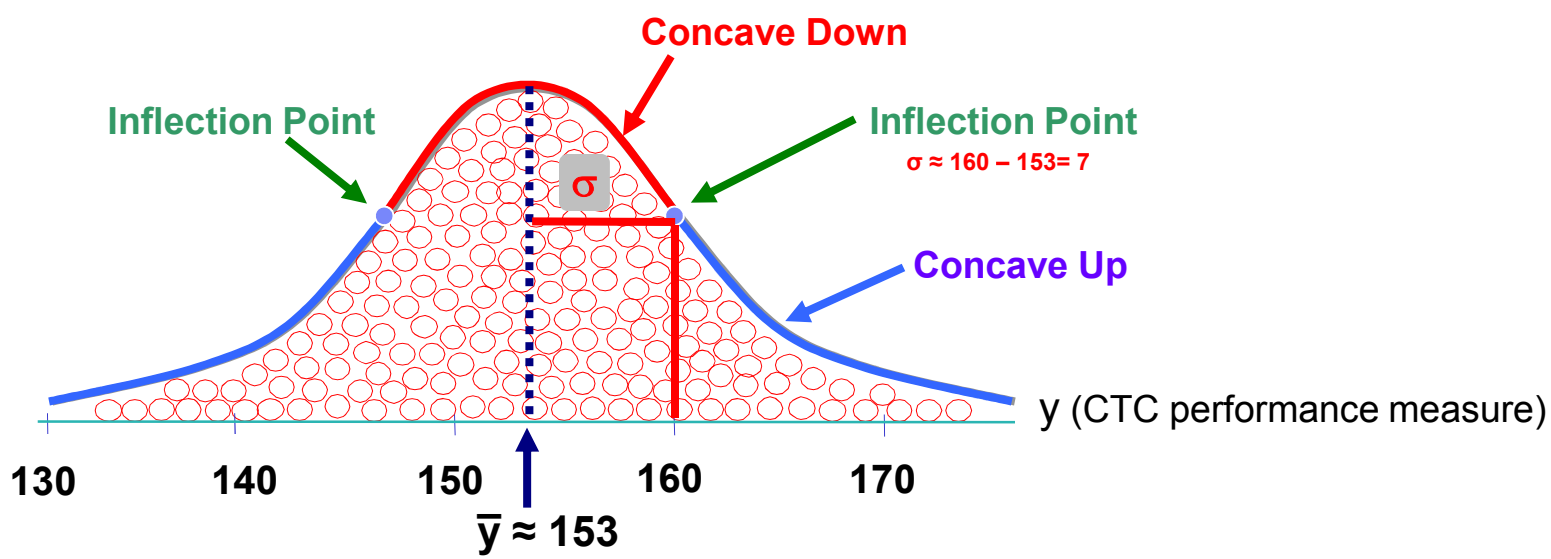
Graphical Meaning of $\bar{y}$ and σ

$\bar{y}$ = **Average = Mean = Balance Point**

σ = **Standard Deviation**



$\sigma \approx$ **average distance of points from the centerline**

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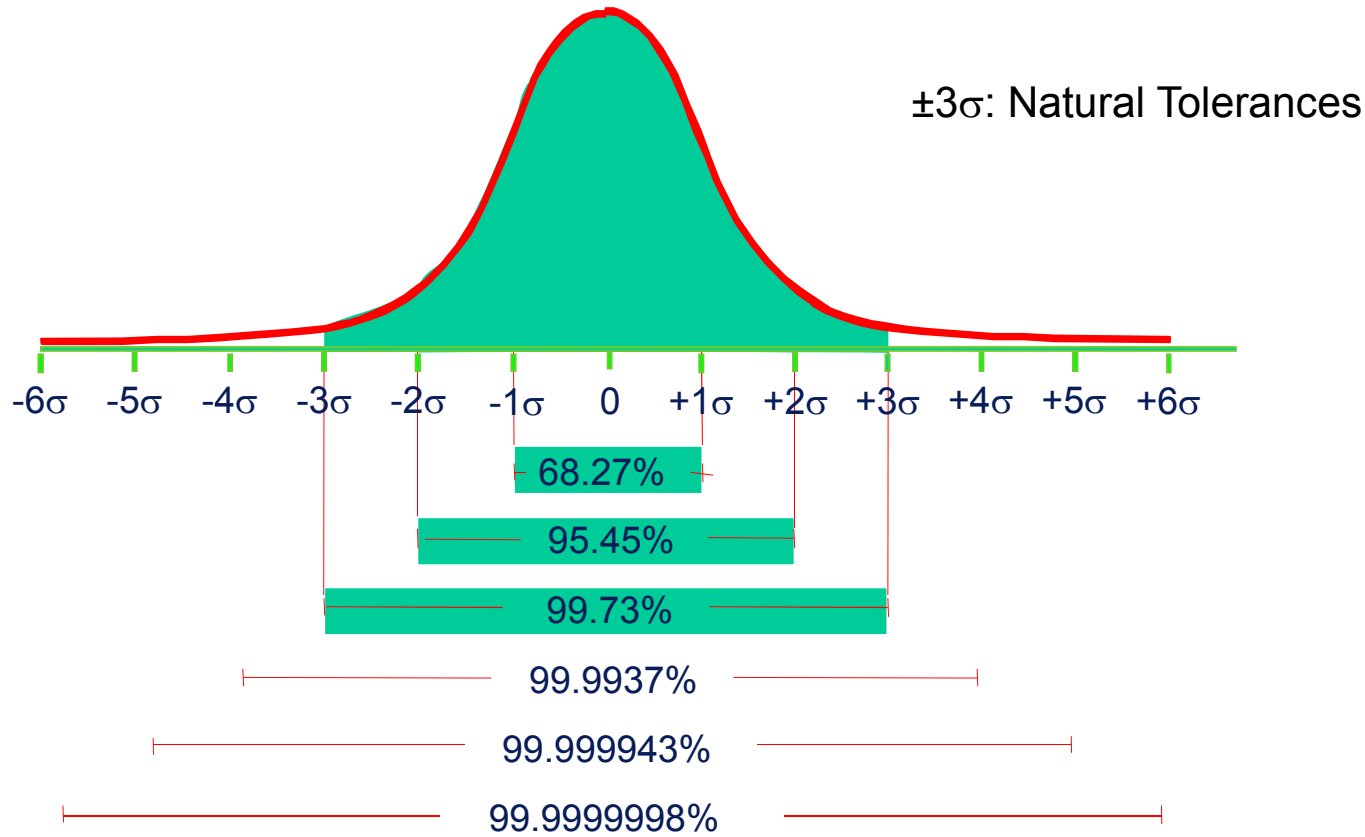
Graphical View of Variation



Typical Areas under the Normal Curve

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Approaches to Testing Multiple Factors

- **Traditional Approaches**
  - One Factor at a Time (OFAT)
  - Oracle (Best Guess)
  - All possible combinations (full factorial)

- **Modern Approach**
  - Statistically designed experiments (DOE) … full factorial plus other selected DOE designs, depending on the situation

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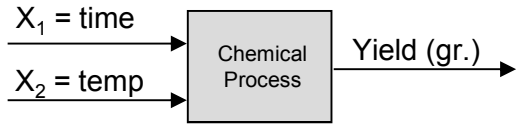
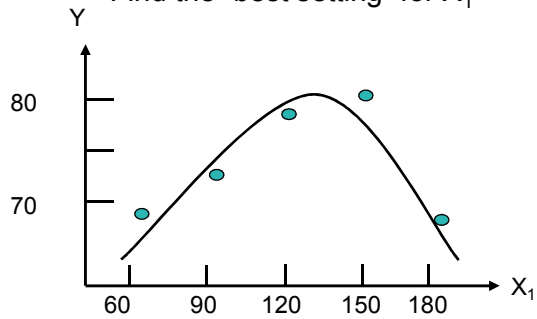
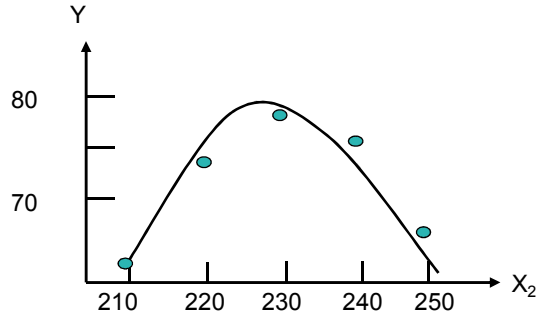
OFAT (One Factor at a Time)



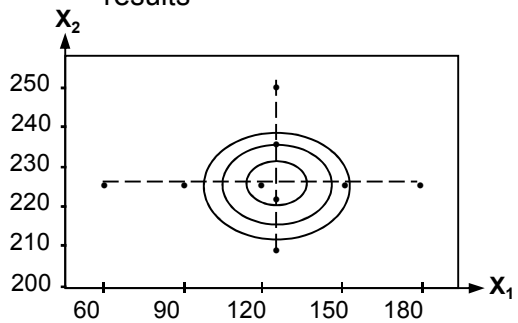
1. Hold X_2 constant and vary X_1
   Find the "best setting" for X_1



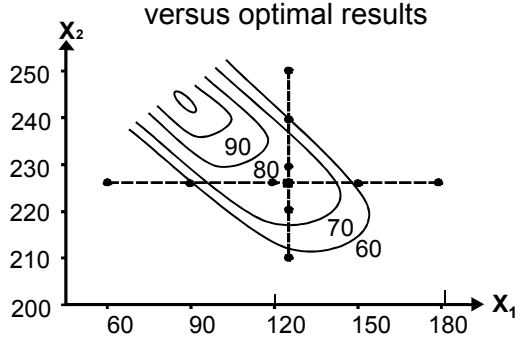
2. Hold X_1 constant at "best setting" and vary X_2.
   Find the "best setting" for X_2.



3. One factor at a time results



4. One factor at a time results versus optimal results



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The Good and Bad about OFAT

- **Good News**
  - Simple
  - Intuitive
  - The way we were originally taught
- **Bad News**
  - Will not be able estimate variable interaction effects
  - Will not be able to generate prediction models and thus not be able to optimize performance

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Oracle (Best Guess)

X1 = W = Wetting Agent (1=.07 ml; 2=none)

X2 = P = Plasticizer (1=1ml; 2=none)

X3 = E = Environment (1=Ambient Mixing; 2=Semi-Evacuated)

X4 = C = Cement (1=Portland Type III; 2=Calcium Aluminate)

X5 = A = Additive (1=No Reinforcement; 2=Steel)

Y = Strength of Lunar Concrete

Run	W	P	E	C	A	Y
1	1	2	1	1	1	5
2	1	1	1	1	1	6
3	2	2	1	1	1	5
4	2	1	1	1	2	6
5	1	2	2	2	2	7
6	1	1	2	2	2	8
7	2	2	2	2	2	10
8	2	1	2	2	1	11

Does factor C shift the average of Y?

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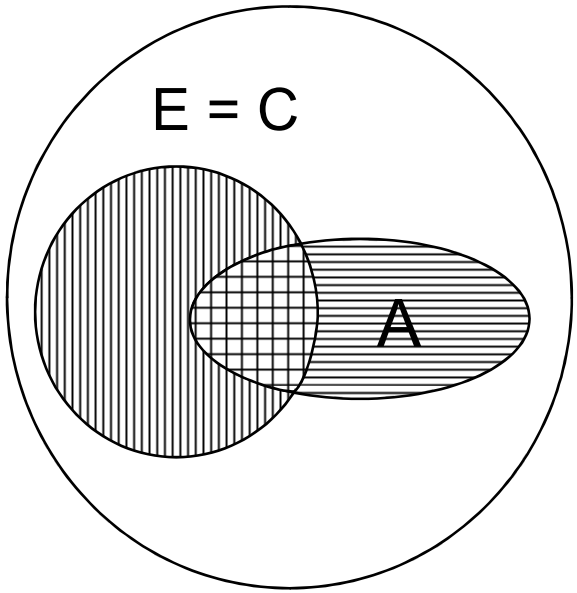
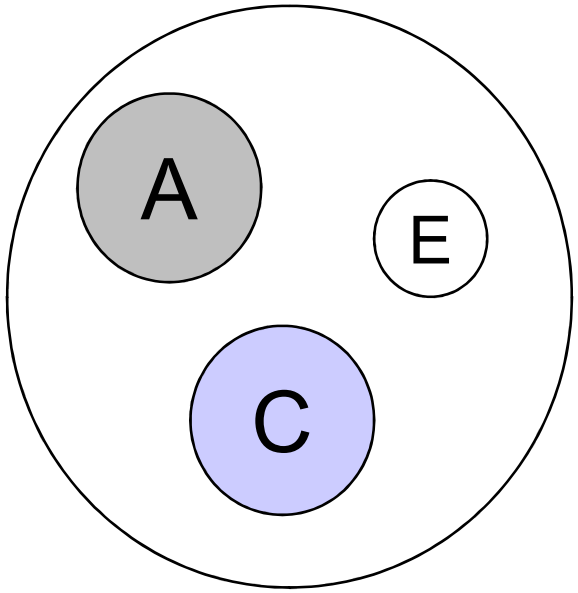
Evaluating the Effects of Variables on Y

What we have is:



What we need is a design to provide independent estimates of effects:



How do we obtain this independence of variables?

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All Possible Combinations (Full Factorial)

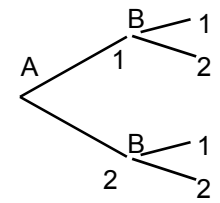
MATRIX FORM

Example 1:

A (2 levels)
B (2 levels)

A	B
1	1
1	2
2	1
2	2

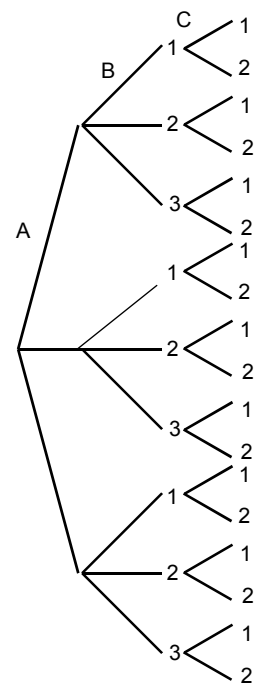
TREE DIAGRAM



Example 2:

A (3 levels)
B (3 levels)
C (2 levels)

A	B	C
1	1	1
1	2	1
1	3	1
2	1	1
2	2	1
2	3	1
3	1	1
3	2	1
3	3	1
1	1	2
1	2	2
1	3	2
2	1	2
2	2	2
2	3	2
3	1	2
3	2	2
3	3	2



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Design of Experiments (DOE)

- An optimal data collection methodology
- "Interrogates" the process
- Used to identify important relationships between input and output factors
- Identifies important interactions between process variables
- Can be used to optimize a process
- Changes "I think" to "I know"

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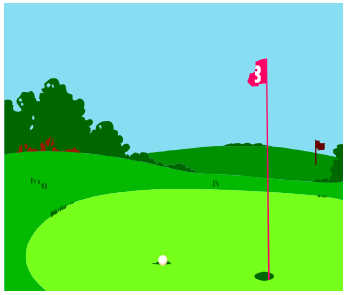
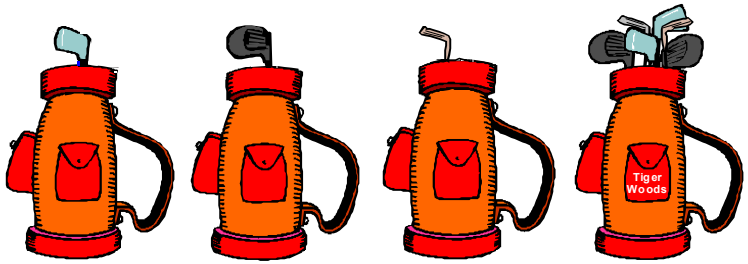
Important Contributions From:

	TAGUCHI	SHAININ	CLASSICAL	BLENDED APPROACH
Loss Function	*			*
Emphasis on Variance Reduction	*			*
Robust Designs	*			*
KISS	*	*		*
Simple Significance Tests		*		*
Component Swapping		*		
Multivariate Charts		*		*
Modeling			*	*
Sample Size			*	*
Efficient Designs			*	*
Optimization			*	*
Confirmation	*			*
Response Surface Methods			*	*





Which bag would a world class golfer prefer?

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Statistically Designed Experiments (DOE):
Orthogonal or Nearly Orthogonal Designs

- FULL FACTORIALS (for small numbers of factors)
- FRACTIONAL FACTORIALS
- PLACKETT - BURMAN
- LATIN SQUARES
- HADAMARD MATRICES

} Taguchi Designs

- BOX - BEHNKEN DESIGNS
- CENTRAL COMPOSITE DESIGNS
- NEARLY ORTHOGONAL LATIN HYPERCUBE DESIGNS

SIMPLE DEFINITION OF TWO-LEVEL ORTHOGONAL DESIGNS

Run	Actual Settings			Coded Matrix			Responses
	(5, 10)	(70, 90)	(100, 200)	(A) Time	(B) Temp	(C) Press	
	A: Time	B: Temp	C: Press				
1	5	70	100	-1	-1	-1	
2	5	70	200	-1	-1	+1	
3	5	90	100	-1	+1	-1	
4	5	90	200	-1	+1	+1	
5	10	70	100	+1	-1	-1	
6	10	70	200	+1	-1	+1	
7	10	90	100	+1	+1	-1	
8	10	90	200	+1	+1	+1	

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The Beauty of Orthogonality:
independent evaluation of effects

A Full Factorial Design for 3 Factors, Each at 2 Levels

Run	A	B	C	AB	AC	BC	ABC
1	-	-	-	+	+	+	-
2	-	-	+	+	-	-	+
3	-	+	-	-	+	-	+
4	-	+	+	-	-	+	-
5	+	-	-	-	-	+	+
6	+	-	+	-	+	-	-
7	+	+	-	+	-	-	-
8	+	+	+	+	+	+	+

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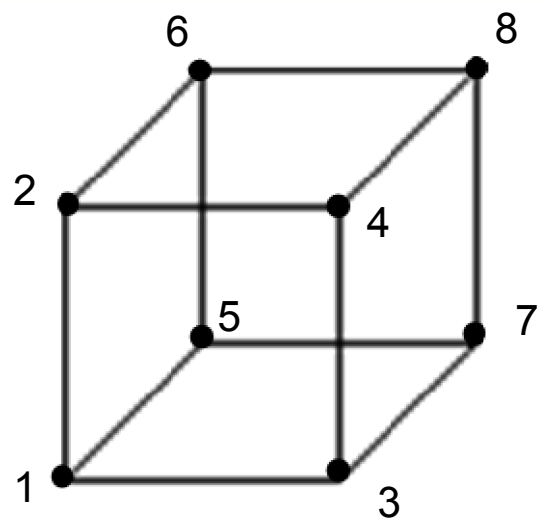
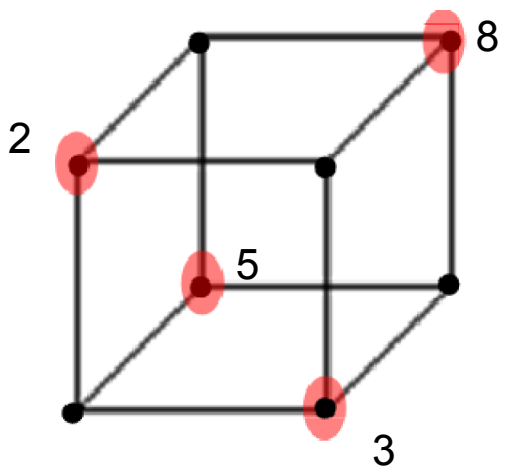
Full Factorial vs. Fractional Factorial

(3 factors at 2 levels)



2^3 = 8-run Full Factorial Design



2^{3-1} = 4-run Fractional Factorial Design

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Screening Design

Taguchi L_{12} Design											
Run	1	2	3	4	5	6	7	8	9	10	11
1	-	-	-	-	-	-	-	-	-	-	-
2	-	-	-	-	-	+	+	+	+	+	+
3	-	-	+	+	+	-	-	-	+	+	+
4	-	+	-	+	+	-	+	+	-	-	+
5	-	+	+	-	+	+	-	+	-	+	-
6	-	+	+	+	-	+	+	-	+	-	-
7	+	-	+	+	-	-	+	+	-	+	-
8	+	-	+	-	+	+	+	-	-	-	+
9	+	-	-	+	+	+	-	+	+	-	-
10	+	+	+	-	-	-	-	+	+	-	+
11	+	+	-	+	-	+	-	-	-	+	+
12	+	+	-	-	+	-	+	-	+	+	-

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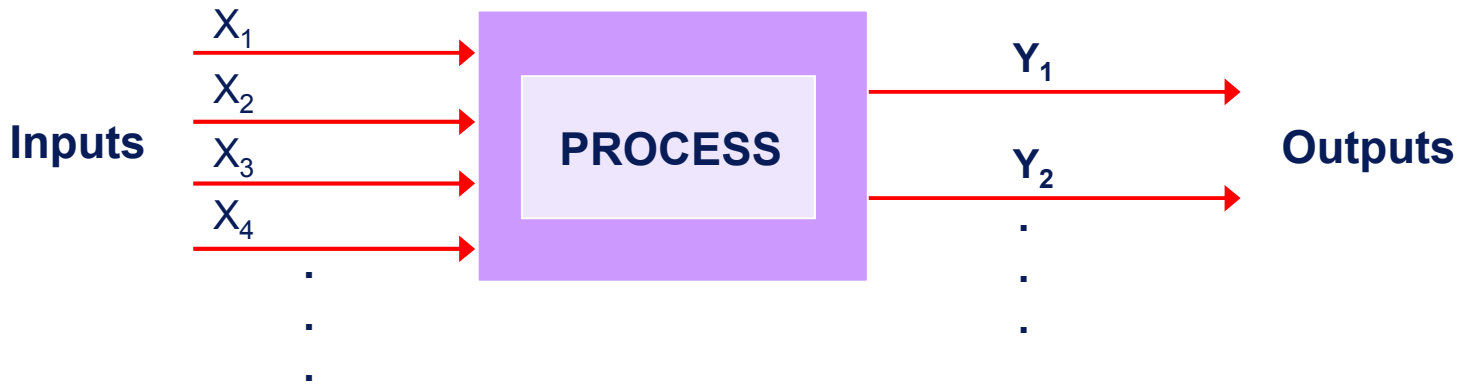
The Purpose of a Designed Experiment

Purposeful changes of the inputs (factors) in order to observe corresponding changes in the output (response).



Run	X_1 X_2 X_3 X_4	Y_1 Y_2	$\bar{Y}$	S_Y
1				
2				
3				
.				
.				

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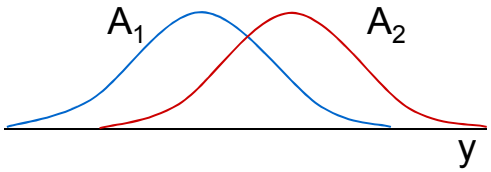
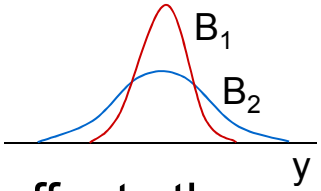
DOE Helps Determine How Inputs Affect Outputs

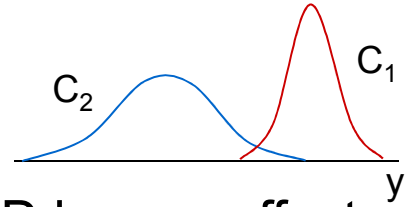
i) Factor A affects the average of y



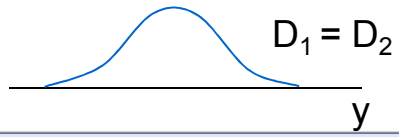
ii) Factor B affects the standard deviation of y



iii) Factor C affects the average and the standard deviation of y



iv) Factor D has no effect on y



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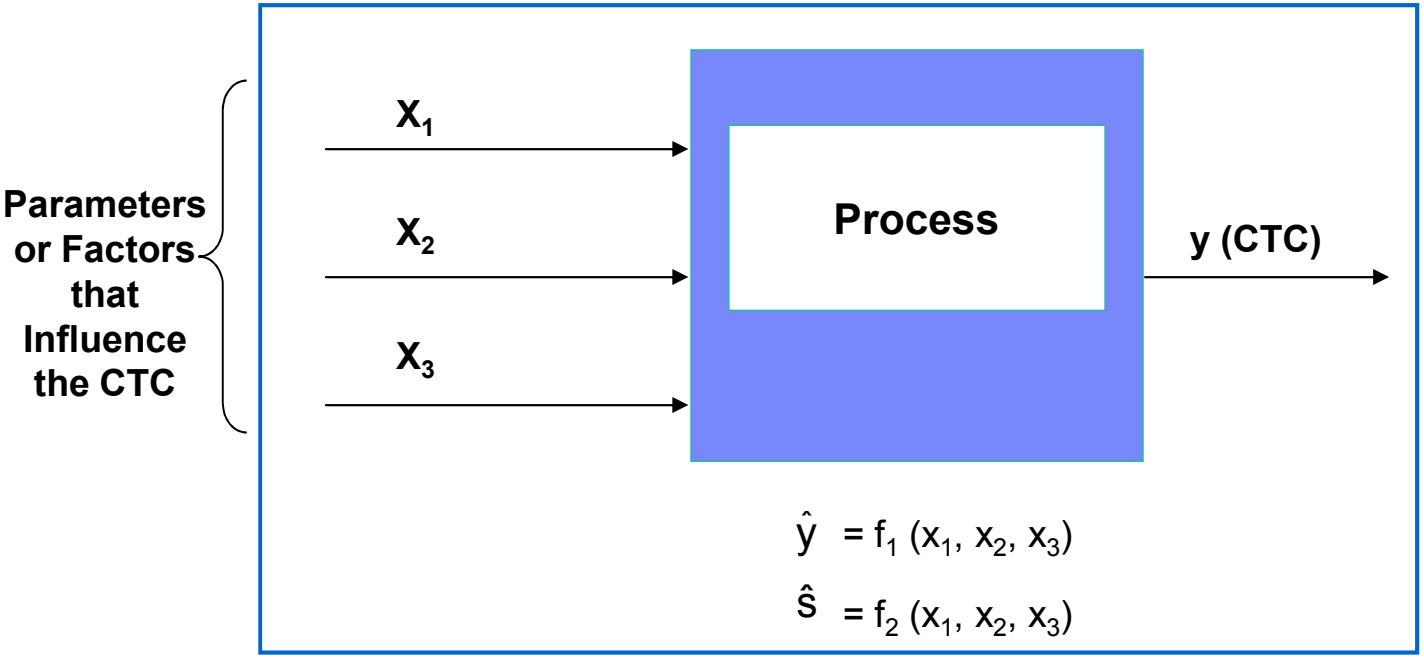
Transfer Functions



Where does the transfer function come from?

- **Exact transfer Function**
- **Approximations**
  - **DOE**
  - **Historical Data Analysis**
  - **Simulation**

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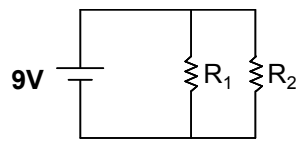
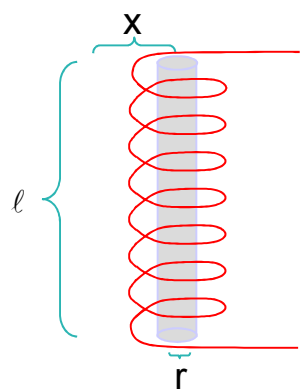
Exact Transfer Functions

- Engineering Relationships
  - V = IR
  - F = ma



The equation for current (I) through this DC circuit is defined by:

$$I = \frac{V}{\frac{R_1 \cdot R_2}{R_1 + R_2}} = \frac{V(R_1 + R_2)}{R_1 \cdot R_2}$$



The equation for magnetic force at a distance X from the center of a solenoid is:

$$H = \frac{NI}{2\ell}\left[\frac{.5\ell + x}{\sqrt{r^2 + (.5\ell + x)^2}} + \frac{.5\ell - x}{\sqrt{r^2 + (.5\ell - x)^2}}\right]$$

Where
- N : total number of turns of wire in the solenoid
- I : current in the wire, in amperes
- r : radius of helix (solenoid), in cm
- ℓ : length of the helix (solenoid), in cm
- x : distance from center of helix (solenoid), in cm
- H : magnetizing force, in amperes per centimeter

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Hierarchical Transfer Functions

$$Y = \text{Gross Margin} = \frac{\text{Gross Profit}}{\text{Gross Revenue}}$$

$$Y = f(y_1, y_2, y_3, y_4, y_5, y_6)$$

$$= \frac{\overset{y_1}{(Rev_{equip}} - \overset{y_2}{COG}) + \overset{y_3}{(Rev_{post\ sales}} - \overset{y_4}{Cost_{post\ sales}}) + \overset{y_5}{(Rev_{fin}} - \overset{y_6}{Cost_{fin}})}{y_1 + y_3 + y_5}$$

$$\overset{y_4}{Cost_{post\ sales}} = f(\overset{x_1}{\text{field cost}}, \overset{x_2}{\text{remote services}}, \overset{x_3}{\text{suppliers}})$$

$$x_1 = f(\text{direct labor, freight, parts, depreciation})$$

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Catapulting Power into Test and Evaluation

Statapult® Catapult

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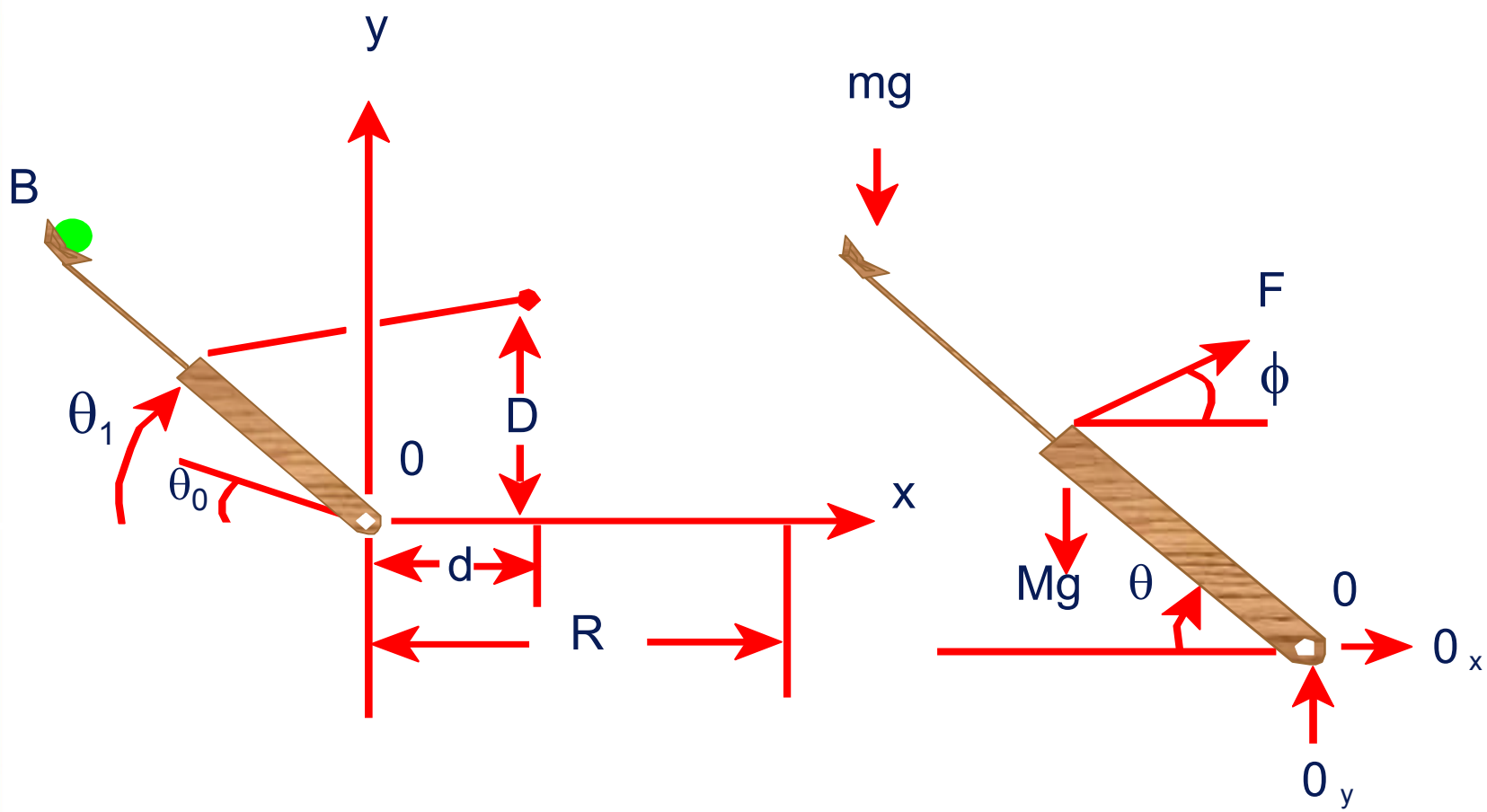
The Theoretical Approach



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The Theoretical Approach (cont.)

$$I_0\ddot{\theta} = r_F F(\theta)\sin\theta\cos\varphi - (Mgr_G + mgr_B)\sin\theta \qquad \tan\phi = \frac{D - r_F\sin\theta}{d + r_F\cos\theta},$$

$$\frac{1}{2}I_0\dot{\theta}^2 = r_F\int_{\theta_0}^{\theta} F(\theta)\sin\theta\cos\varphi\, d\theta - (Mgr_G + mgr_B)(\sin\theta - \sin\theta_0)$$

$$\frac{1}{2}I_0\dot{\theta}_1^2 = r_F\int_{\theta_0}^{\theta_1} F(\theta)\sin\theta\cos\varphi\, d\theta - (Mgr_G + mgr_B)(\sin\theta_1 - \sin\theta_0).$$

$$x = v_B\cos\left(\frac{\pi}{2} - \theta_1\right)t - \frac{1}{2}r_B\cos\theta_1 \qquad y = r_B\sin\theta_1 + v_B\sin\left(\frac{\pi}{2} - \theta_1\right)t - \frac{1}{2}gt^2.$$

$$r_B\sin\theta_1 + (R + r_B\cos\theta_1)\tan\left(\frac{\pi}{2} - \theta_1\right) - \frac{g}{2V_B^2}\frac{(R + r_B\cos\theta_1)^2}{\cos^2\left(\frac{\pi}{2} - \theta_1\right)} = 0.$$

$$\frac{gI_0}{4r_B}\frac{(R + r_B\cos\theta_1)^2}{\cos^2\left(\frac{\pi}{2} - \theta_1\right)\left[r_B\sin\theta_1 + (R + r_B\cos\theta_1)\tan\left(\frac{\pi}{2} - \theta_1\right)\right]}$$

$$= r_F\int_{\theta_0}^{\theta_1} F(\theta)\sin\theta\cos\phi\, d\theta - (Mgr_G + mgr_B)(\sin\theta_1 - \sin\theta_0).$$

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Statapult® DOE Demo
(The Empirical Approach)

	Actual Factors		Coded Factors			Response Values			
Run	A	B	A	B	AB	Y_1	Y_2	$\bar{Y}$	S
1	144	2	-1	-1	+1				
2	144	3	-1	+1	-1				
3	160	2	+1	-1	-1				
4	160	3	+1	+1	+1				
		Avg –							
		Avg +							
		Δ							

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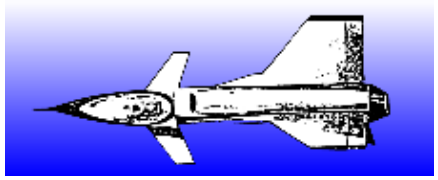
Value Delivery: Reducing Time to Market for New Technologies



INPUT **OUTPUT**

Pitch <) (0, 15, 30)

Roll <) (0, 15, 30)

W1F <) (-15, 0, 15)

W2F <) (-15, 0, 15)

W3F <) (-15, 0, 15)

Modeling Flight Characteristics of New 3-Wing Aircraft

Six Aero-Characteristics

- **Total # of Combinations = 3^5 = 243**
- **Central Composite Design: n = 30**

Patent Holder: Dr. Bert Silich

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Aircraft Equations

C_L = .233 + .008(P)^2 + .255(P) + .012(R) - .043(WD1) - .117(WD2) + .185(WD3) + .010(P)(WD3) - .042(R)(WD1) + .035(R)(WD2) + .016(R)(WD3) + .010(P)(R) - .003(WD1)(WD2) - .006(WD1)(WD3)

C_D = .058 + .016(P)^2 + .028(P) - .004(WD1) - .013(WD2) + .013(WD3) + .002(P)(R) - .004(P)(WD1) - .009(P)(WD2) + .016(P)(WD3) - .004(R)(WD1) + .003(R)(WD2) + .020(WD1)^2 + .017(WD2)^2 + .021(WD3)^2

C_Y = -.006(P) - .006(R) + .169(WD1) - .121(WD2) - .063(WD3) - .004(P)(R) + .008(P)(WD1) - .006(P)(WD2) - .008(P)(WD3) - .012(R)(WD1) - .029(R)(WD2) + .048(R)(WD3) - .008(WD1)^2

C_M = .023 - .008(P)^2 + .004(P) - .007(R) + .024(WD1) + .066(WD2) - .099(WD3) - .006(P)(R) + .002(P)(WD2) - .005(P)(WD3) + .023(R)(WD1) - .019(R)(WD2) - .007(R)(WD3) + .007(WD1)^2 - .008(WD2)^2 + .002(WD1)(WD2) + .002(WD1)(WD3)

C_{YM}= .001(P) + .001(R) - .050(WD1) + .029(WD2) + .012(WD3) + .001(P)(R) - .005(P)(WD1) - .004(P)(WD2) - .004(P)(WD3) + .003(R)(WD1) + .008(R)(WD2) - .013(R)(WD3) + .004(WD1)^2 + .003(WD2)^2 - .005(WD3)^2

C_e = .003(P) + .035(WD1) + .048(WD2) + .051(WD3) - .003(R)(WD3) + .003(P)(R) - .005(P)(WD1) + .005(P)(WD2) + .006(P)(WD3) + .002(R)(WD1)

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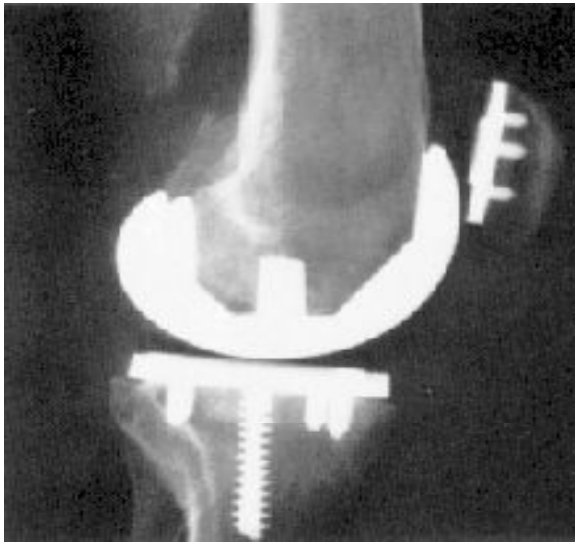
Fusing Titanium and Cobalt-Chrome



Courtesy Rai Chowdhary

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DOE "Market Research" Example

Suppose that, in the auto industry, we would like to investigate the following automobile attributes (i.e., factors), along with accompanying levels of those attributes:

A: Brand of Auto:	-1 = foreign		+1 = domestic
B: Auto Color:	-1 = light	0 = bright	+1 = dark
C: Body Style:	-1 = 2-door	0 = 4-door	+1 = sliding door/hatchback
D: Drive Mechanism:	-1 = rear wheel	0 = front wheel	+1 = 4-wheel
E: Engine Size:	-1 = 4-cylinder	0 = 6-cylinder	+1 = 8-cylinder
F: Interior Size:	-1 ≤ 2 people	0 = 3-5 people	+1 ≥ 6 people
G: Gas Mileage:	-1 ≤ 20 mpg	0 = 20-30 mpg	+1 ≥ 30 mpg
H: Price:	-1 ≤ $20K	0 = $20-$40K	+1 ≥ $40K

In addition, suppose the respondents chosen to provide their preferences to product profiles are taken based on the following demographic:

J: Age:	-1 ≤ 25 years old	+1 ≥ 35 years old
K: Income:	-1 ≤ $30K	+1 ≥ $40K
L: Education:	-1 < BS	+1 ≥ BS

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DOE "Market Research" Example (cont.)

Question: **Choose the best design for evaluating this scenario**

Answer: **L_{18} design with attributes A - H in the inner array and factors J, K, and L in the outer array, resembling an L_{18} robust design, as shown below:**

									L	-	+	-	+	-	+	-	+		
									K	-	-	+	+	-	-	+	+		
									J	-	-	-	-	+	+	+	+		
Run*	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**		y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	$\bar{y}$	**s**
1	-	-	-	-	-	-	-	-											
2	-	-	0	0	0	0	0	0											
3	-	-	+	+	+	+	+	+											
4	-	0	-	-	0	0	+	+											
5	-	0	0	0	+	+	-	-											
6	-	0	+	+	-	-	0	0											
7	-	+	-	0	-	+	0	+											
8	-	+	0	+	0	-	+	-											
9	-	+	+	-	+	0	-	0											
10	+	-	-	+	+	0	0	-											
11	+	-	0	-	-	+	+	0											
12	+	-	+	0	0	-	-	+											
13	+	0	-	0	+	-	+	0											
14	+	0	0	+	-	0	-	+											
15	+	0	+	-	0	+	0	-											
16	+	+	-	+	0	+	-	0											
17	+	+	0	-	+	-	0	+											
18	+	+	+	0	-	0	+	-											

Segmentation of the population or

Respondent Profiles

* 18 different product profiles

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Modeling The Drivers of Turnover

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Google on DOE
(quotes* from Daryl Pregibon, Google Engineer)

"From a user's perspective, a query was submitted and results appear. From Google's perspective, the user has provided an opportunity to test something. What can we test? Well, there is so much to test that we have an Experiment Council that vets experiment proposals and quickly approves those that pass muster."

" We evangelize experimentation to the extent that we provide a mechanism for advertisers to run their own experiments.

. . . allows an advertiser to run a (full) factorial experiment on its web page. Advertisers can explore layout and content alternatives while Google randomly directs queries to the resulting treatment combinations. Simple analysis of click and conversion rates allows advertisers to explore a range of alternatives and their effect on user awareness and interest."

*** Taken From: *Statistics @ Google* in Amstat News, May 2009**

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DOE: the bridge to Monte Carlo Simulation and Design Optimization

- **Expected Value Analysis**
- **Parameter (Robust) Design**
- **Tolerance Allocation**

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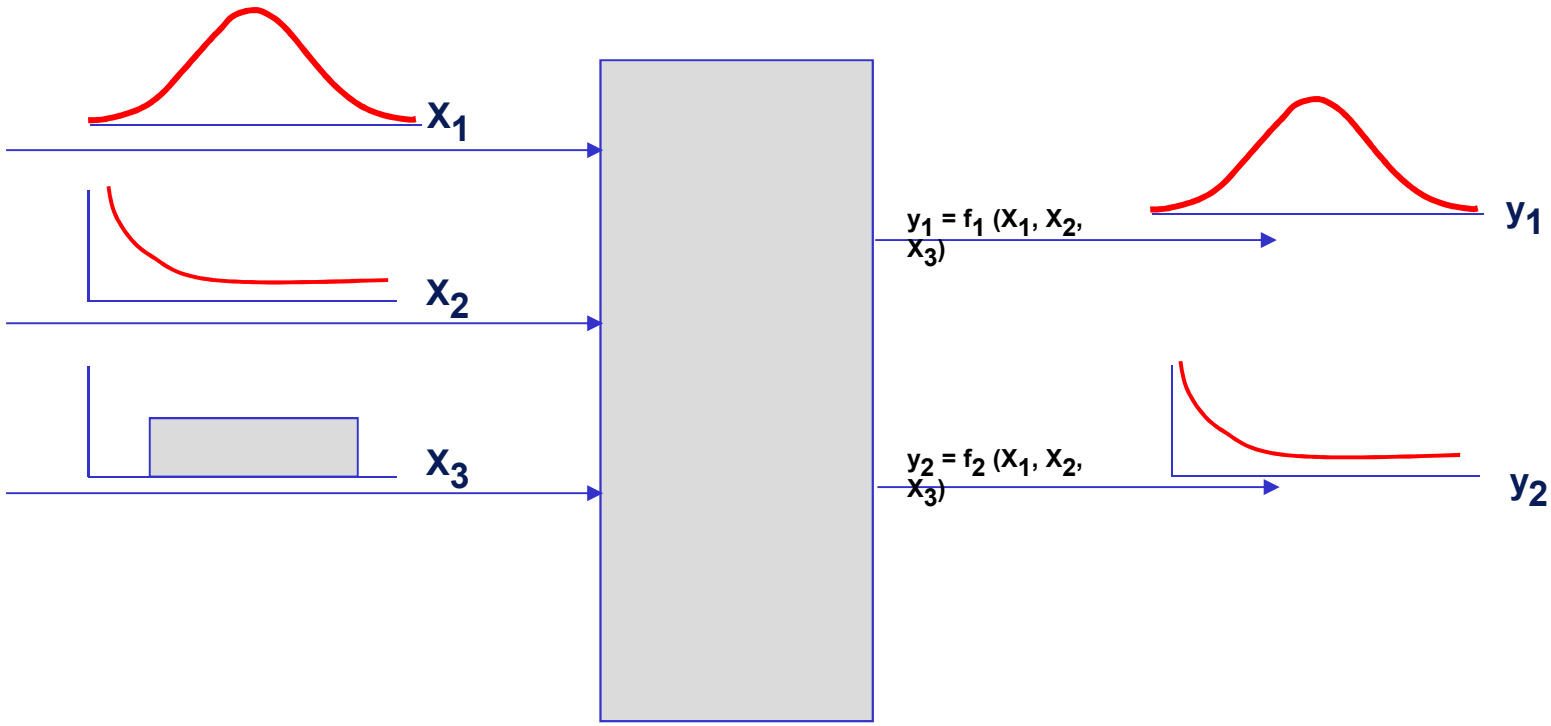
Expected Value Analysis (EVA)

EVA is the technique used to determine the characteristics of the output distribution (mean, standard deviation, and shape) when we have knowledge of (1) the input variable distributions and (2) the transfer functions.



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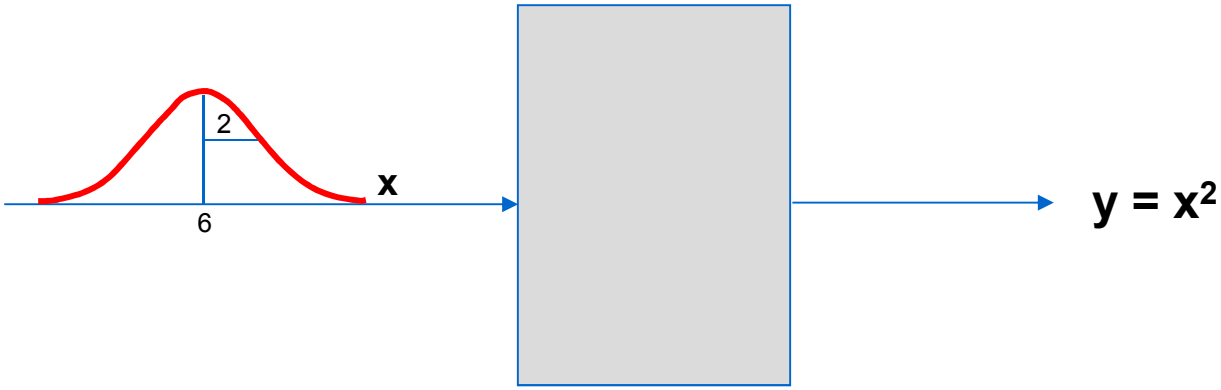
Expected Value Analysis Example



What is the mean or expected value of the y distribution?

What is the shape of the y distribution?

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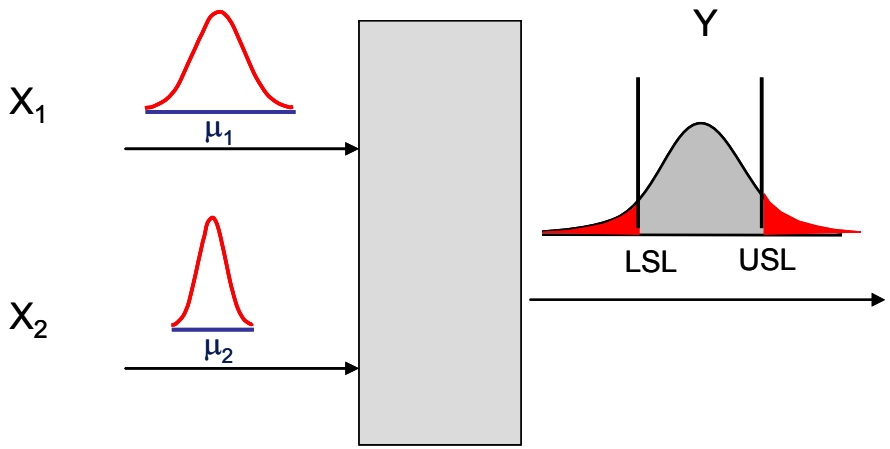
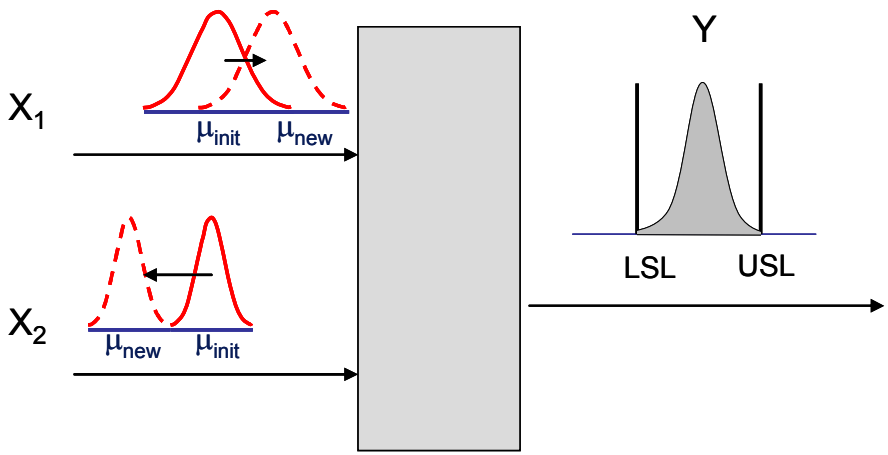
Parameter Design (Robust Design)



Process of finding the optimal mean settings of the input variables to minimize the resulting dpm.



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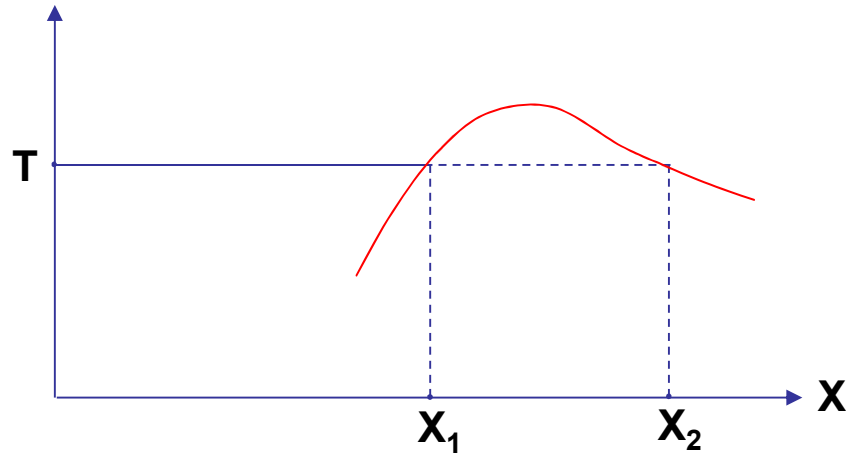
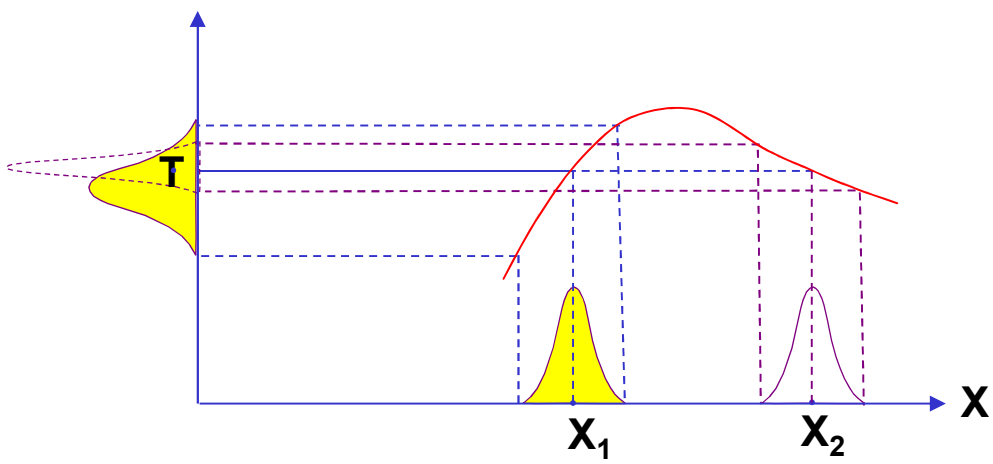
Parameter Design (Robust Design)



If you're the designer, which setting for X do you prefer?



Changing the mean of an input may possibly reduce the output variation!

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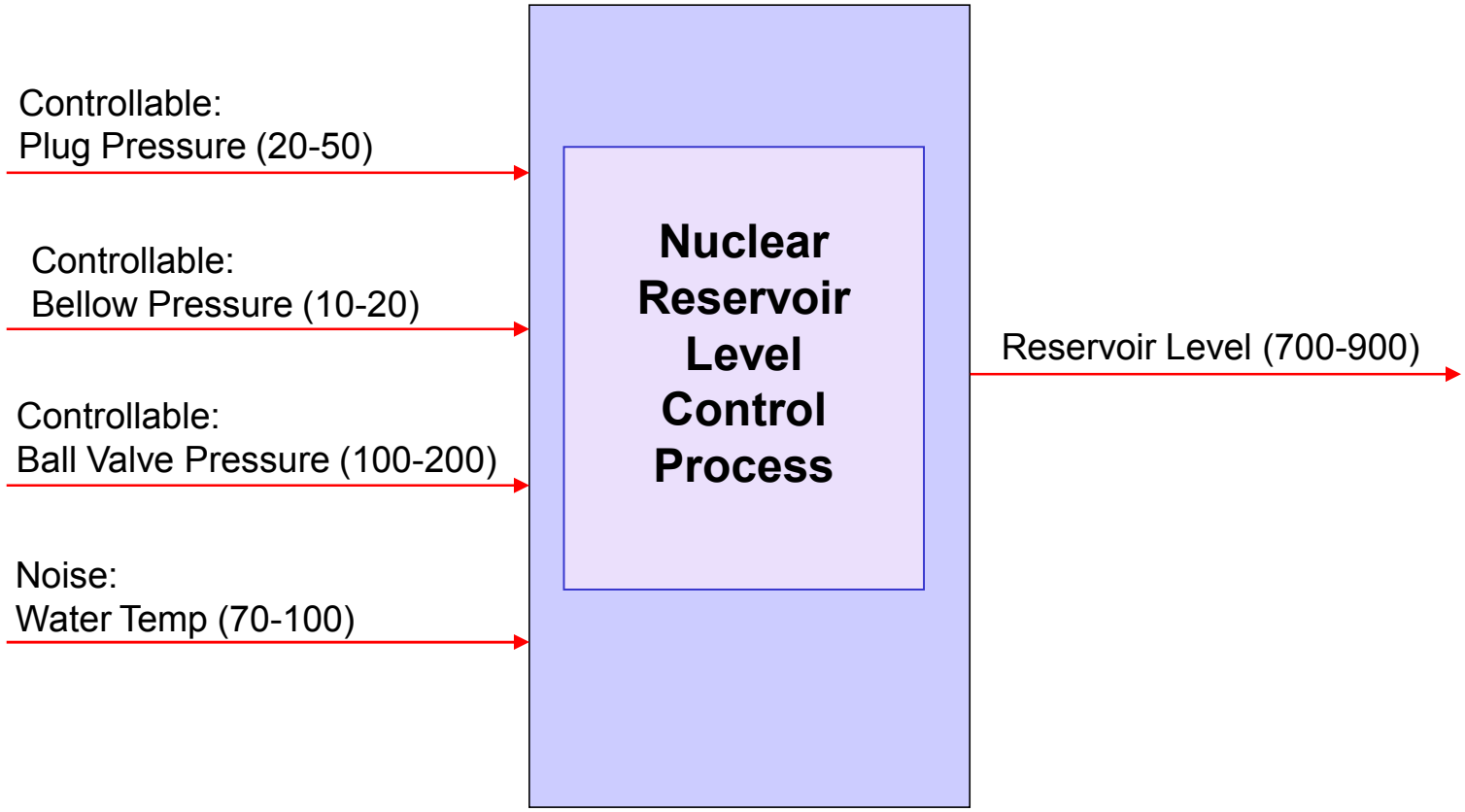
Robust (Parameter) Design Simulation* Example



* From **SimWare Pro** by Digital Computations

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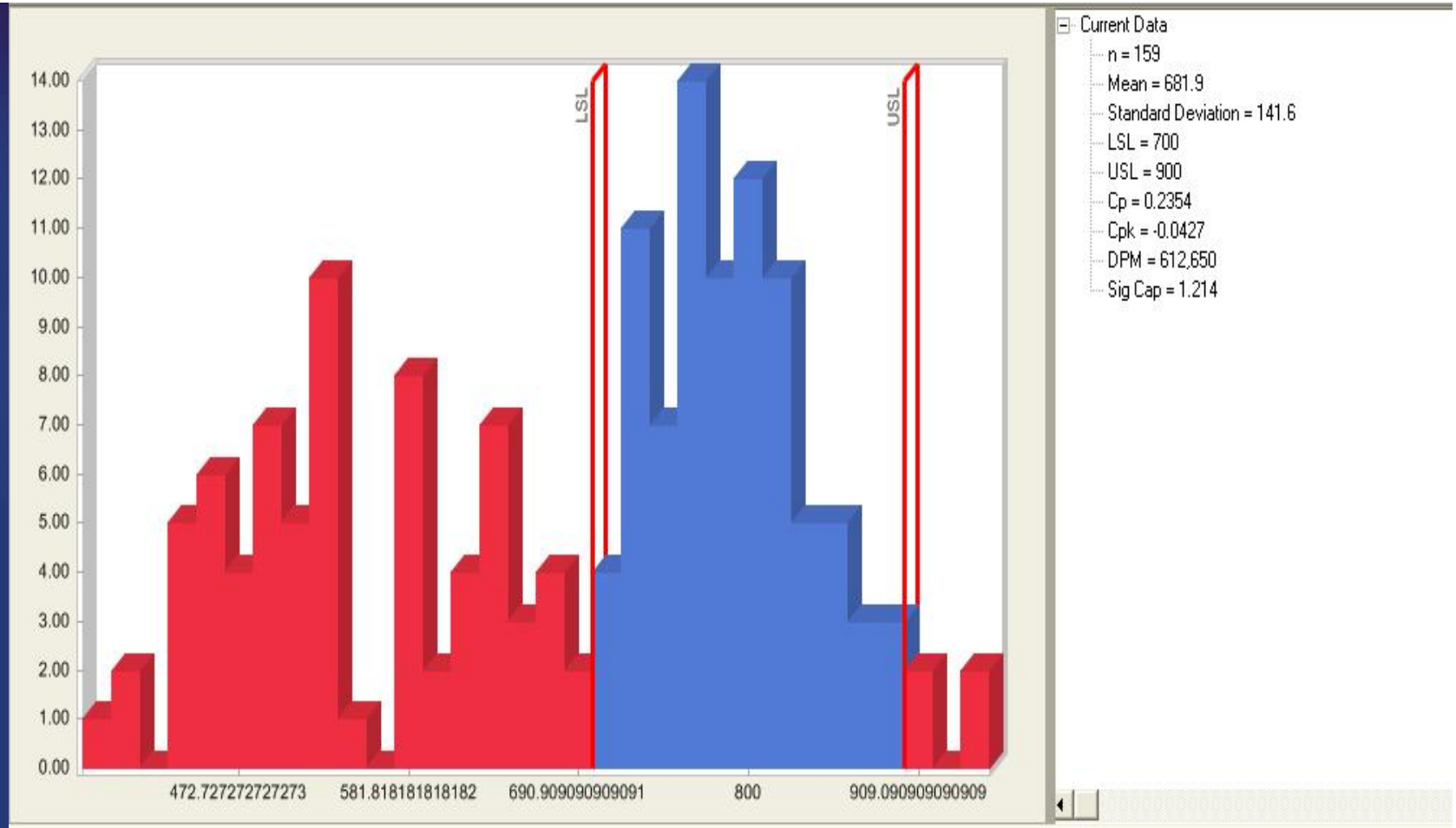

Input Controls

Control Set 1

50.0 — Plug Pressure (20 to 50)

12.00 — Bellow Pressure (10 to 20)

145 — Ball Valve Pressure (100 to 200)

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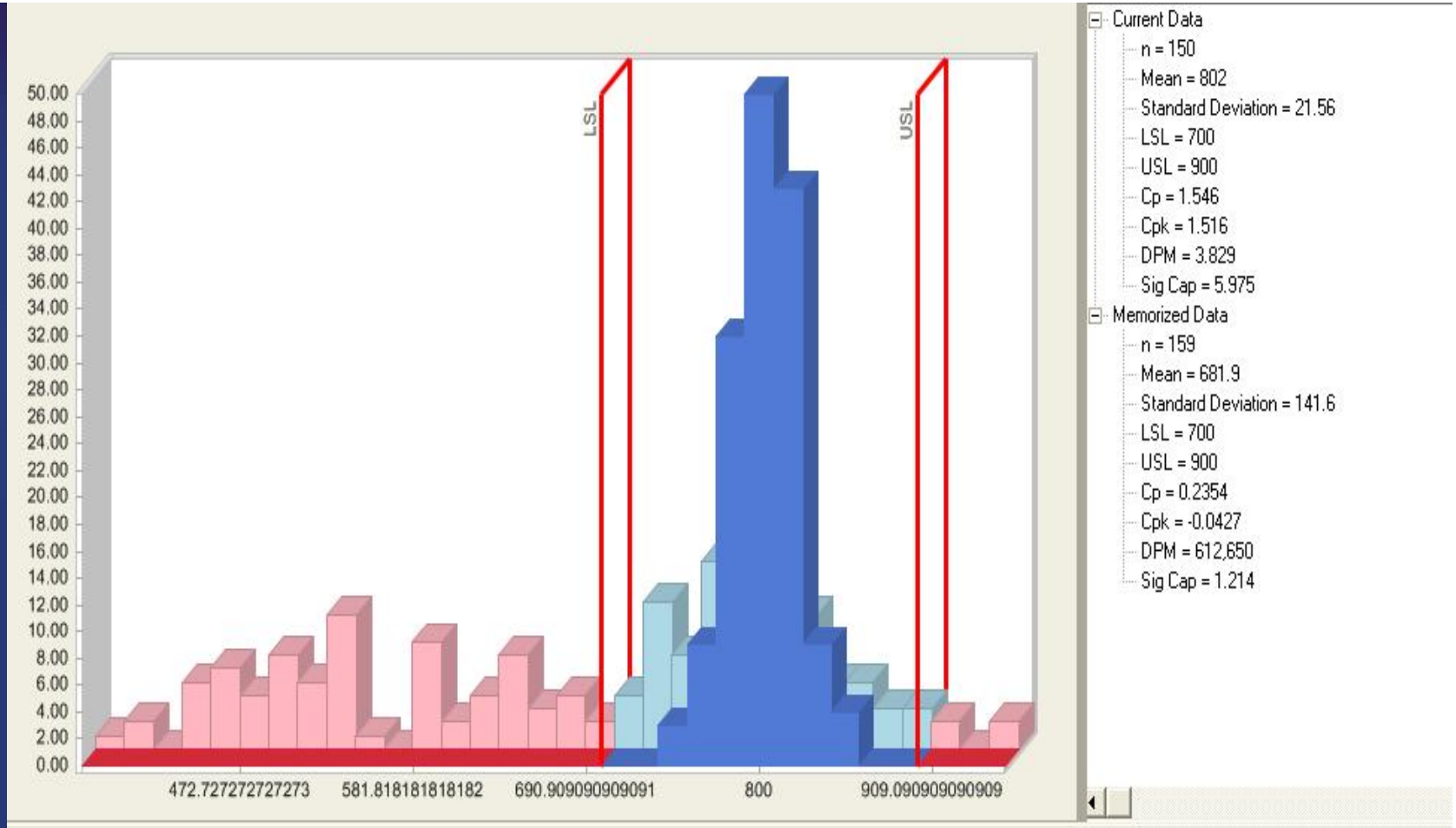

Input Controls

Control Set 1

21.0 — Plug Pressure (20 to 50)

20.00 — Bellow Pressure (10 to 20)

137 — Ball Valve Pressure (100 to 200)

Water Temp (Expensive to Control)

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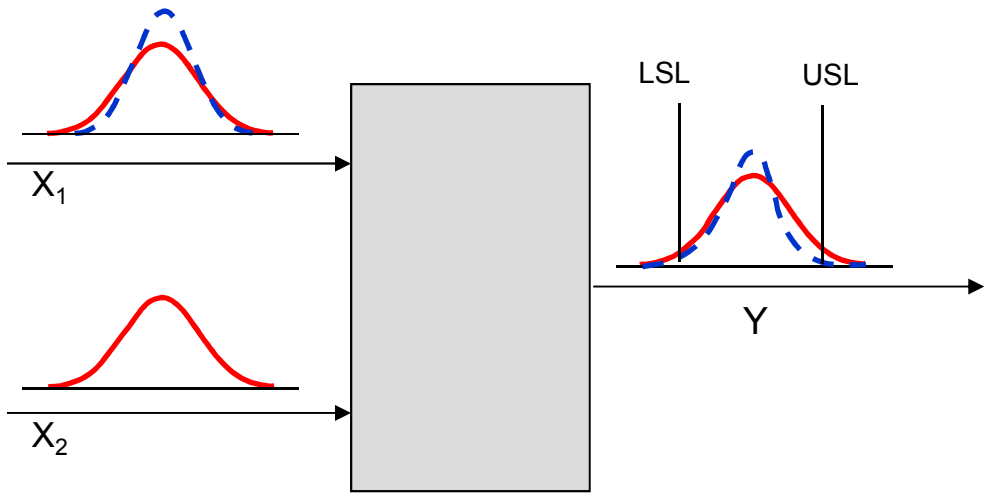
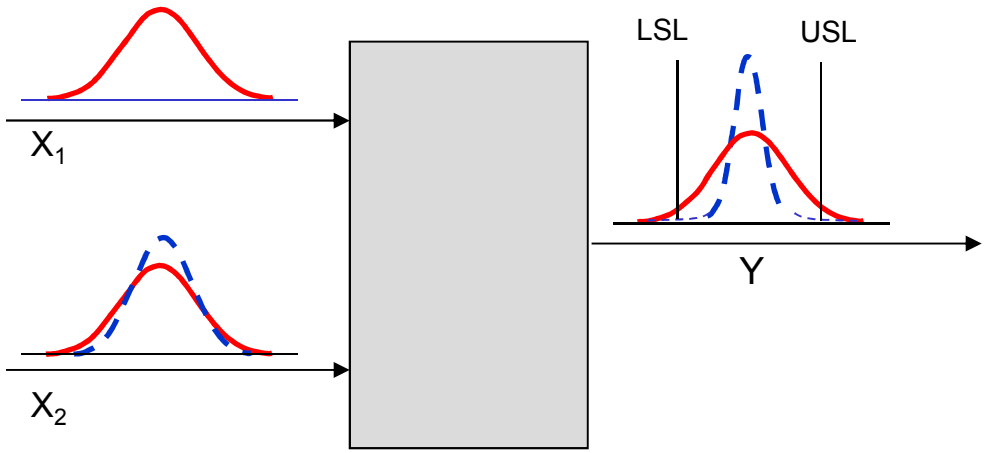
Tolerance Allocation



Which input standard deviations have the biggest effect on the output variation?



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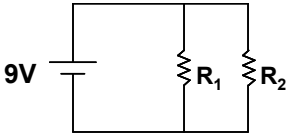
Expected Value Analysis Example

In the simple DC circuit shown below, Ohm's Law says that the total current (I) is equal to V / R, where R is the equivalent resistance of the network.



For this circuit, $$I = \frac{V}{\frac{R_1 \cdot R_2}{R_1 + R_2}} = \frac{V(R_1 + R_2)}{R_1 \cdot R_2}$$

Suppose

- **R_1 is normally distributed with a mean of 50 Ohms and a standard deviation of 2 Ohms**
- **R_2 is normally distributed with a mean of 100 Ohms and standard deviation of 4 Ohms**
- **the specification limits for current (I) are LSL = .255A and USL = .285A**

what is the capability of I?
what current distribution is expected?

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Expected Value Analysis Example (cont.)

Expected Value Analysis

Process Inputs			
Factor	Distro	First Parameter	Second Parameter
R1	Normal	50	2
R2	Normal	100	4

Process Outputs	current
# of Simulations	1,000,000
Mean	0.2704
StdDev	0.0081
Median	0.2702
LSL	0.255
USL	0.285
Normal Distro Statistics	
KS Test p-Value (Normal)	NA
dpm	65,091.811
Cpk	0.598
Cp	0.616
Sigma Level	1.793
Sigma Capability	3.013
Observed Defect Statistics	
Actual defects	64,780
dpm	64,780.0
95% Conf. Inv Lower	64,298.341
95% Conf. Inv Upper	65,264.182

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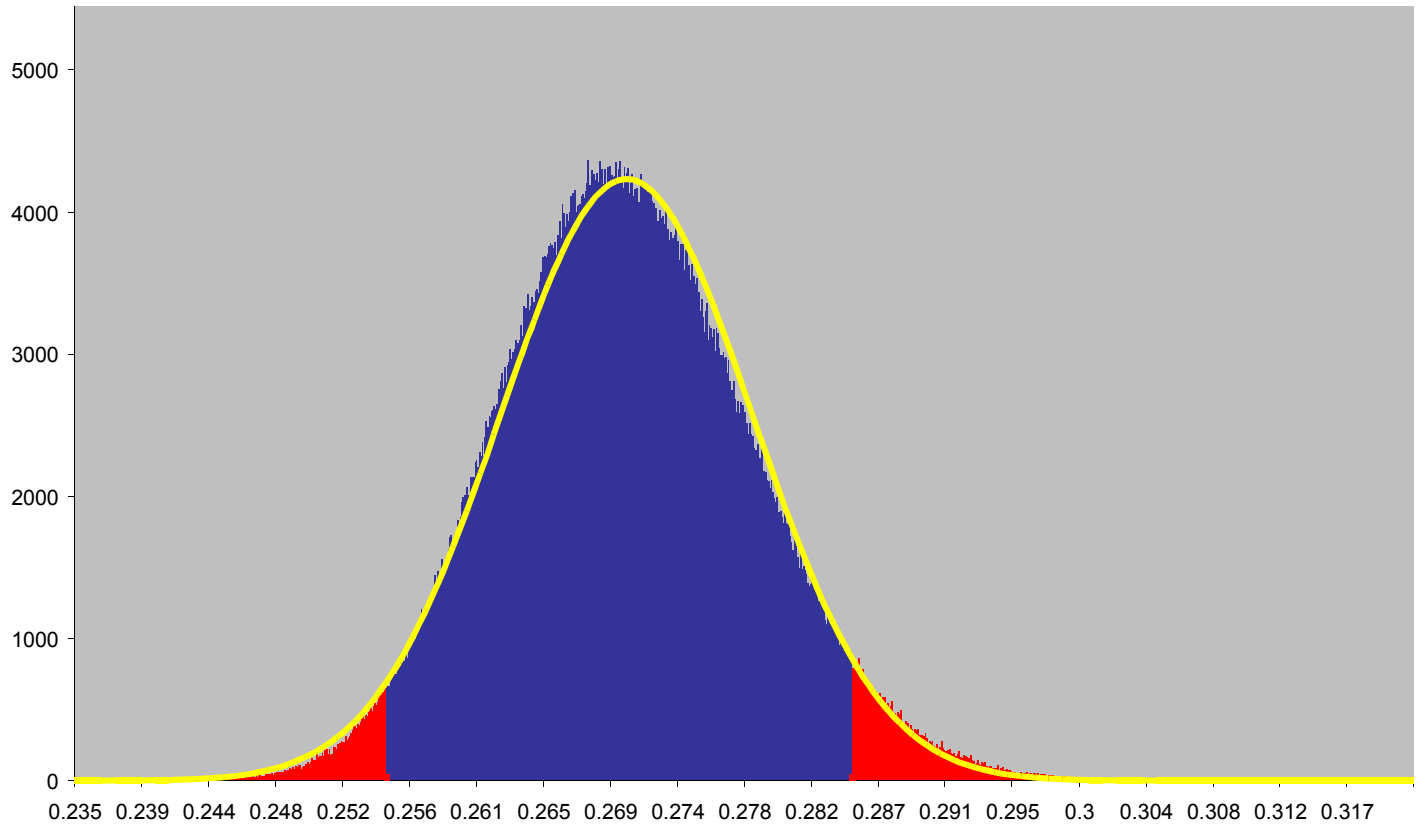
Expected Value Analysis Example (cont.)

Normal Distribution
Mean = 0.2704
Std Dev = 0.00812

Histogram of current (amps)



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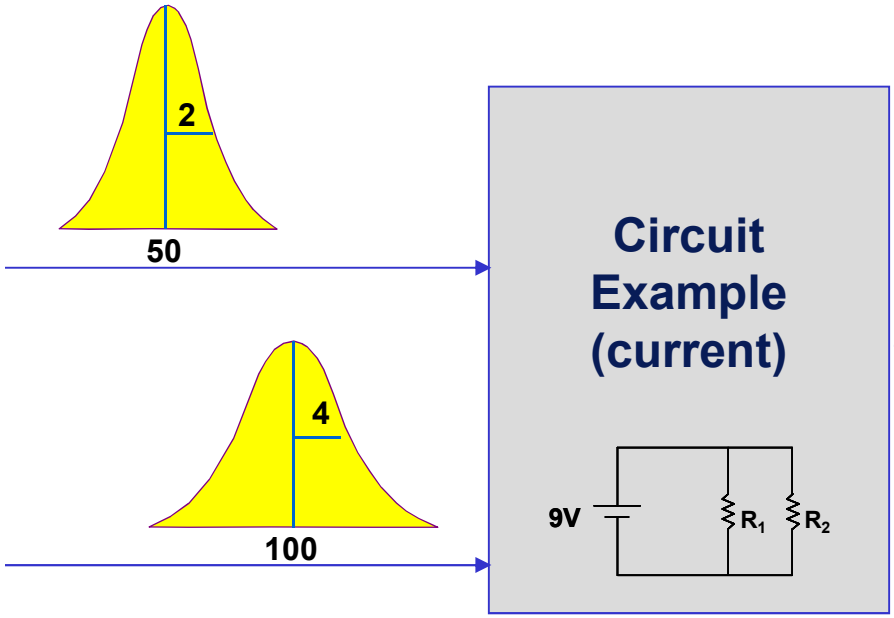
Tolerance Allocation Example



$$I = \frac{9(R_1 + R_2)}{R_1 \cdot R_2}$$

LSL = .255

USL = .285

Which resistor's standard deviation has the greater impact on the capability of I?

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Tolerance Allocation Example (cont.)

A reduction in R_1's standard deviation (sigma) significantly reduces the dpm while a reduction in R_2's standard deviation has a smaller effect.

Tolerance Allocation Table

Process Inputs			
Factor	Distro	First Parameter	Second Parameter
R1	Normal	50	2
R2	Normal	100	4

N = 10,000 (in dpm)		
current Table (Normal dpm)		
	R1	R2
-50% Sigma	2,897	45,852
-25% Sigma	21,912	53,427
-10% Sigma	46,150	58,483
Nominal	63,975	63,438
+10% Sigma	88,478	69,198
+25% Sigma	127,102	83,522
+50% Sigma	196,089	100,553

A reduction in R_1's standard deviation by 50% (from 2 ohms to 1 ohm) combined with an increase in R_2's standard deviation by 25% (from 4 ohms to 5 ohms) results in a dpm = 9,743.

(This result is not shown in the table.)

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Growth Rate of Factorial Designs

For 2-level designs and k factors: 2^k combinations

- for k = 2 factors: 2^2 = 4 combinations
- for k = 3 factors: 2^3 = 8 combinations
- for k = 10 factors: 2^{10} = 1,024 combinations

For 3-level designs and k factors: 3^k combinations

- for k = 2 factors: 3^2 = 9 combinations
- for k = 3 factors: 3^3 = 27 combinations
- for k = 10 factors: 3^{10} = 59,049 combinations

What if the # of factors and/or the number of levels gets large?

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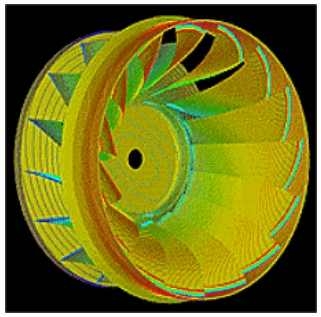
Examples of Simulation and High Performance Computing (HPC)

Power

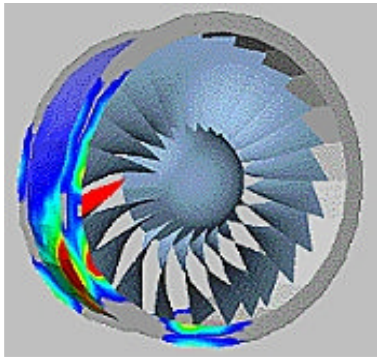


Simulation of stress and vibrations of turbine assembly for use in nuclear power generation

Automotive

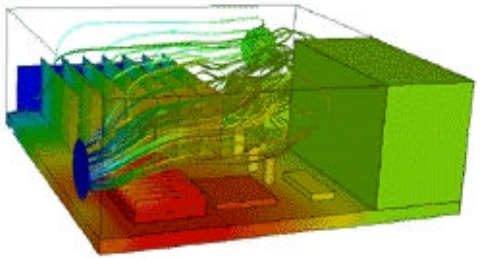
Simulation of underhood thermal cooling for decrease in engine space and increase in cabin space and comfort

Aerospace



Evaluation of dual bird-strike on aircraft engine nacelle for turbine blade containment studies

Electronics



Evaluation of cooling air flow behavior inside a computer system chassis

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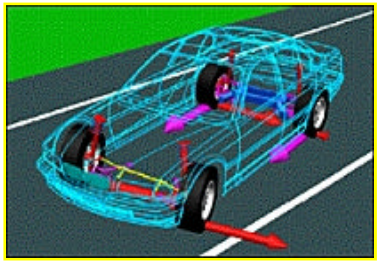
Examples of Computer Aided Engineering (CAE) and Simulation Software

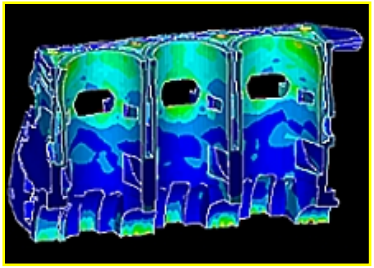
Mechanical motion: **Multibody kinetics and dynamics**

- **ADAMS®**
- **DADS**



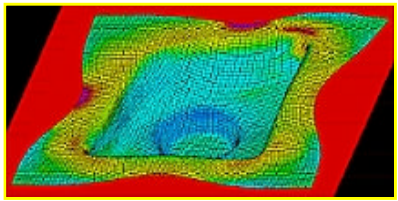
Implicit Finite Element Analysis: **Linear and nonlinear statics, dynamic response**

- **MSC.Nastran™, MSC.Marc™**
- **ANSYS®**
- **Pro MECHANICA**
- **ABAQUS® Standard and Explicit**
- **ADINA**



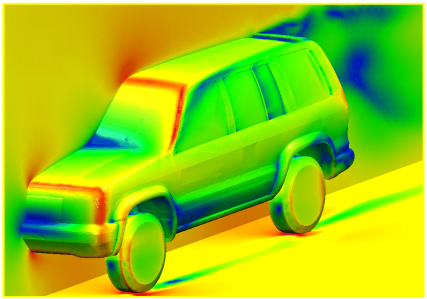
Explicit Finite Element Analysis : **Impact simulation, metal forming**

- **LS-DYNA**
- **RADIOSS**
- **PAM-CRASH®, PAM-STAMP**



General Computational Fluid Dynamics: **Internal and external flow simulation**

- **STAR-CD**
- **CFX-4, CFX-5**
- **FLUENT®, FIDAP™**
- **PowerFLOW®**



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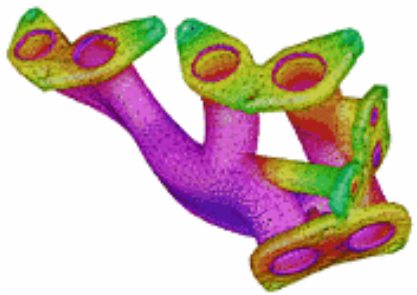
Examples of Computer Aided Engineering (CAE) and Simulation Software (cont.)

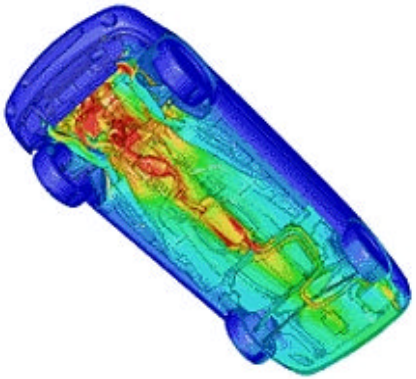
Preprocessing: Finite Element Analysis and Computational Fluid Dynamics mesh generation

- ICEM-CFD
- Gridgen
- Altair® HyperMesh®
- I-deas®
- MSC.Patran
- TrueGrid®
- GridPro
- FEMB
- ANSA



Postprocessing: Finite Element Analysis and Computational Fluid Dynamics results visualization

- Altair® HyperMesh®
- I-deas
- MSC.Patran
- FEMB
- EnSight
- FIELDVIEW
- ICEM CFD Visual3 2.0 (PVS)
- COVISE



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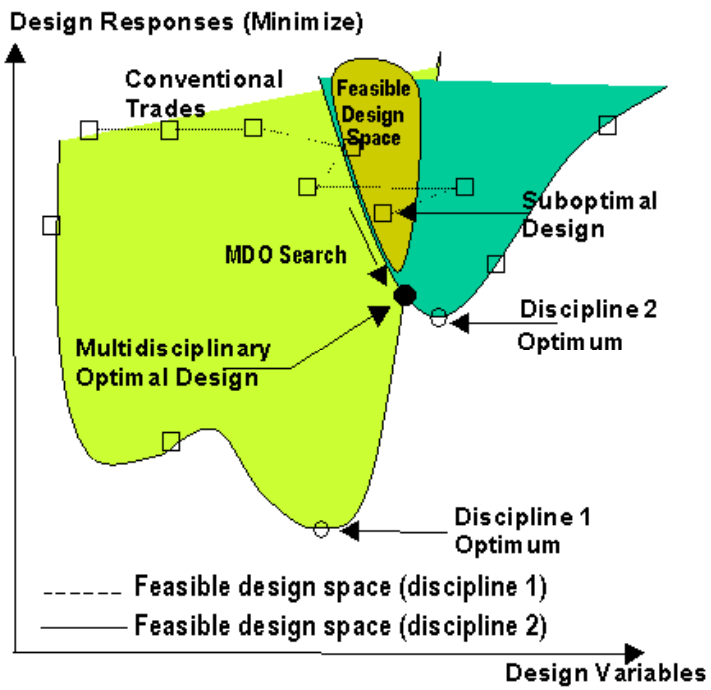
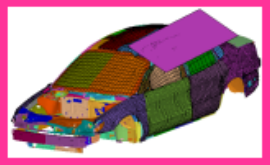
Multidisciplinary Design Optimization (MDO): A Design Process Application





- CFD
- Structures
- Performance
- Controls
- Cost

- NVH
- Safety
- Durability
- Controls Stability
- Cost

Key Elements of MDO

- Massive Computational Problem;
- Solution by decomposition effective for complex systems;
- Multiprocessor computing simplifies MDO solutions conceptually & enables solutions previously intractable;
- Aids in the management of the design process.

Mastery of interactions between the disciplines (or, subsystems) is as important as the methods & tools used within a single discipline

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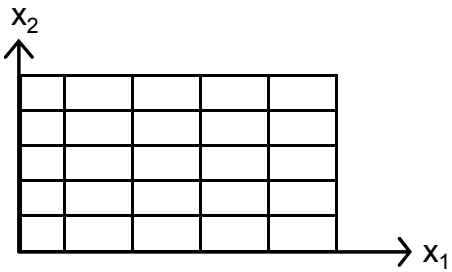
Latin Hypercube Sampling

- Method to populate the design space when using deterministic simulation models or when many variables are involved.

- Design space has k variables (or dimensions).

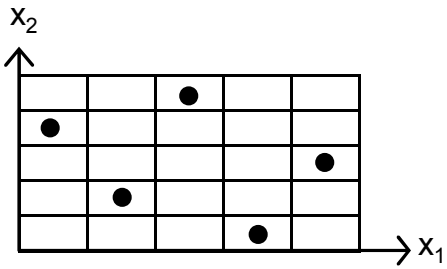
  Ex: Assume k = 2



- Suppose a sample of size n is to be taken; Stratify the design space into n^k cells.

  Ex: Assume n = 5; $n^k = 5^2 = 25$

  Note: there are n=5 strata for each of the k=2 dimensions.



- Each of the n points is sampled such that each marginal strata is represented only once in the sample.

  Note: each sample point has its own unique row and column.

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Applying Modeling and Simulation to Automotive Vehicle Design

IDENTIFY CTCs, CDPs → Many, Many x's → **SCREENING DESIGN (DOE PRO)** → The critical few CDP's

Examples of CTCs:

y_1 = weight of vehicle

y_2 = cost of vehicle

Safety CTCs with constraints specified by FMVSS (Federal Motor Vehicle Safety Standards)

- y_3 = frontal head impact
- y_4 = frontal chest impact
- y_5 = toe board intrusion
- y_6 = hip deflection
- y_7 = rollover impact
- y_8 = side impact

RADIOSS
DYNA
MADYMO

no federal requirements on these CTCs

- y_9 = internal aerodynamics (airflow)
- y_{10} = external aerodynamics (airflow)

CFD

- y_{11} = noise
- y_{12} = vibration (e.g., steering wheel)
- y_{13} = harshness (e.g., over bumps, shocks)
- y_{14} = durability (at 100K miles)

NASTRAN

NASTRAN RADIOSS MADYMO

Integrated processes with high fidelity CAE analyses on HPC servers

Examples of Critical Design Parameters (CDPs or Xs):

x_1 = roof panel material

x_2 = roof panel thickness

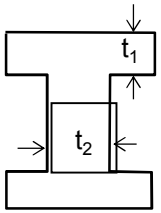
x_3 = door pillar dimensions ⇒ i beam

x_4 = shape/geometry

x_5 = windshield glass

x_6 = hood material, sizing and thickness

x_7 = under hood panel material, sizing and thickness



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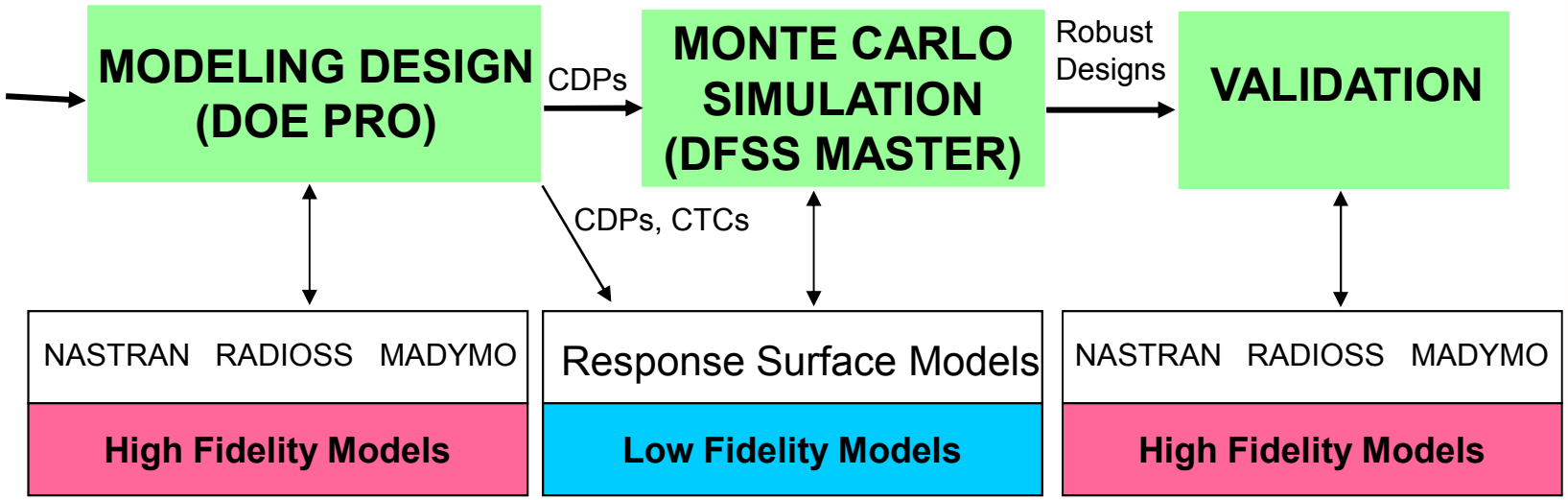
Applying Modeling and Simulation to Automotive Vehicle Design (cont.)



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Summary of "Modeling the Simulator"

Perform Screening Design Using the Simulator if necessary

↓ Critical Parameters ID'd

Perform Modeling Design Using the Simulator to Build Low Fidelity Model

↓ Transfer Function on Critical Parameters

Perform Expected Value Analysis, Robust Design, and Tolerance Allocation Using Transfer Function

↓ Optimized Design

Validate Design Using the Simulator

↓ Optimized Simulator

Build Prototype to Validate Design in Real World

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Environments Where Simulation and Modeling Is Beneficial

- **A high number of design variables**
- **A substantial number of design subsystems and engineering disciplines**
- **Interdependency and interaction between the subsystems and variables**
- **Multiple response variables**
- **Need to characterize the system at a higher level of abstraction**
- **Time and/or space must be compressed**

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Introduction to High Throughput Testing (HTT)

- A recently developed technique based on combinatorics
- Used to test myriad combinations of many factors (typically qualitative) where the factors could have many levels
- Uses a minimum number of runs or combinations to do this
- Software (e.g., ProTest) is needed to select the minimal subset of all possible combinations to be tested so that all 2-way combinations are tested.
- HTT is not a DOE technique, although the terminology is similar
- A run or row in an HTT matrix is, like DOE, a combination of different factor levels which, after being tested, will result in a successful or failed run
- HTT has its origins in the pharmaceutical business where in drug discovery many chemical compounds are combined together (combinatorial chemistry) at many different strengths to try to produce a reaction.
- Other industries are now using HTT, e.g., software testing, materials discovery, integration, and verification testing (see example on next page).

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HTT Example
(Performance Verification and Validation)

- **We would like to perform verification testing with 4 input factors described below.**

- **All possible combinations would involve how many test combinations?**

- **If we were interested in testing all pairs only, how many runs would b in the test matrix and what would those combinations be? To answer this question, we used our ProTest software. See next page.**

Sensor Type	Weapon Type	External Data Link	Target Type
S1	**W1**	**Yes**	**T1**
S2	**W2**	**No**	**T2**
S3	**W3**		**T3**
S4			**T4**
			T5

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High Throughput Testing Example (cont)
20 Test Cases

	Sensor	Weapon	Data Link	Target
Case 1	S1	W2	Yes	T1
Case 2	S4	W1	Yes	T2
Case 3	S2	W1	No	T3
Case 4	S3	W3	Yes	T4
Case 5	S2	W3	Yes	T5
Case 6	S4	W3	No	T1
Case 7	S3	W2	No	T2
Case 8	S1	W3	Yes	T3
Case 9	S1	W1	No	T4
Case 10	S3	W1	No	T5
Case 11	S2	W1	No	T1
Case 12	S1	W3	No	T2
Case 13	S4	W2	No	T3
Case 14	S2	W2	Yes	T4
Case 15	S4	W2	No	T5
Case 16	S3	W2	Yes	T3
Case 17	S1	W1	Yes	T5
Case 18	S2	W2	Yes	T2
Case 19	S3	W3	Yes	T1
Case 20	S4	W2	No	T4

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HTT: Pairwise Testing Optimization

- **Suppose we had 75 Factors to test.**
- **Suppose we wanted to test each of these at 2 levels.**
- **How many total combinations are there?**

  2^{75} = 37, 778, 931, 862, 957, 161, 709, 568

  i.e., 37 Sextillion, 778 Quintillion, 931 Quadrillion, 862 Trillion, 957 Billion, 161 Million, 709 Thousand, 568

- **What is the minimum number of these combinations that will have to be tested in order to test every 2-way combination?**
- **To answer this question, we used our Pro-Test software. The answer is 14 runs or experimental combinations.**
- **For k factors each having the same number of levels tested, say v, then the minimum number of tests ≈ v^2 (ln k)**

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HTT Applications

- **Reducing the cost and time of testing while maintaining adequate test coverage**
- **Integration, functionality, and verification testing**
- **Creating a test plan to stress a product and discover problems**
- **Prescreening before a large DOE to ensure all 2-way combinations are feasible before discovering, midway through an experiment, that certain combinations are not feasible**
- **Developing an "outer array" of noise combinations to use in a robust design DOE when the number of noise factors and settings is large**

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Key Take-Aways

- Various approaches to combinatorial test, to include OFAT and Oracle.
- DOE brings orthogonal or nearly orthogonal designs into play.
- Orthogonality (both vertical and horizontal balance in a design) is key to being able to evaluate the effects of factors and their interactions independently from one another.
- Factorial designs are great, but in a world of large test design spaces, we need something else.
- Nearly Orthogonal Latin Hypercube Designs provide a sampling strategy to test a large number of factors with a much smaller number of runs than what a factorial design requires, while still retaining adequate orthogonality. These are particularly useful when designing experiments for computer simulations.
- High Throughput Testing is a way to get great test coverage (i.e., all pairwise combinations) with a minimal number of runs. This would be a candidate design for OT&E when we are trying to verify and validate performance in an operational envelope.

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