Combinatorial Test Design Methodology

IEEE/Lockheed Martin Webinar
March 13, 2014

Dr. Mark J. Kiemele
Air Academy Associates

Agenda

- **Some Basic Definitions and Terms**
- **Various Approaches to Testing Multiple Factors**
- **Design of Experiments (DOE): a Modern Approach to Combinatorial Testing**
- **Testing in Very Large Design Spaces**
- **High Throughput Testing (All Pairs Testing)**
- **Q&A**

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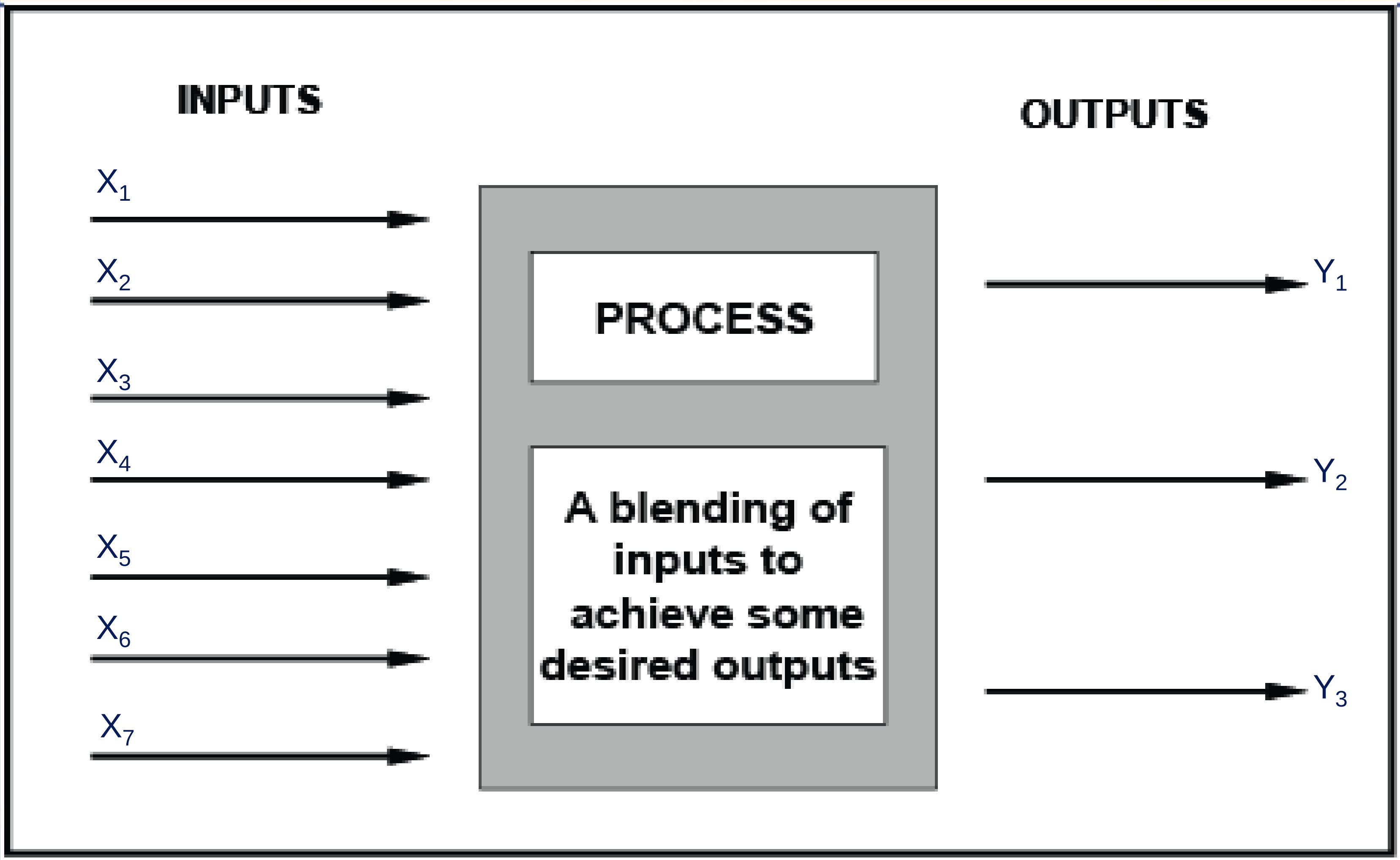
Definition of a Process



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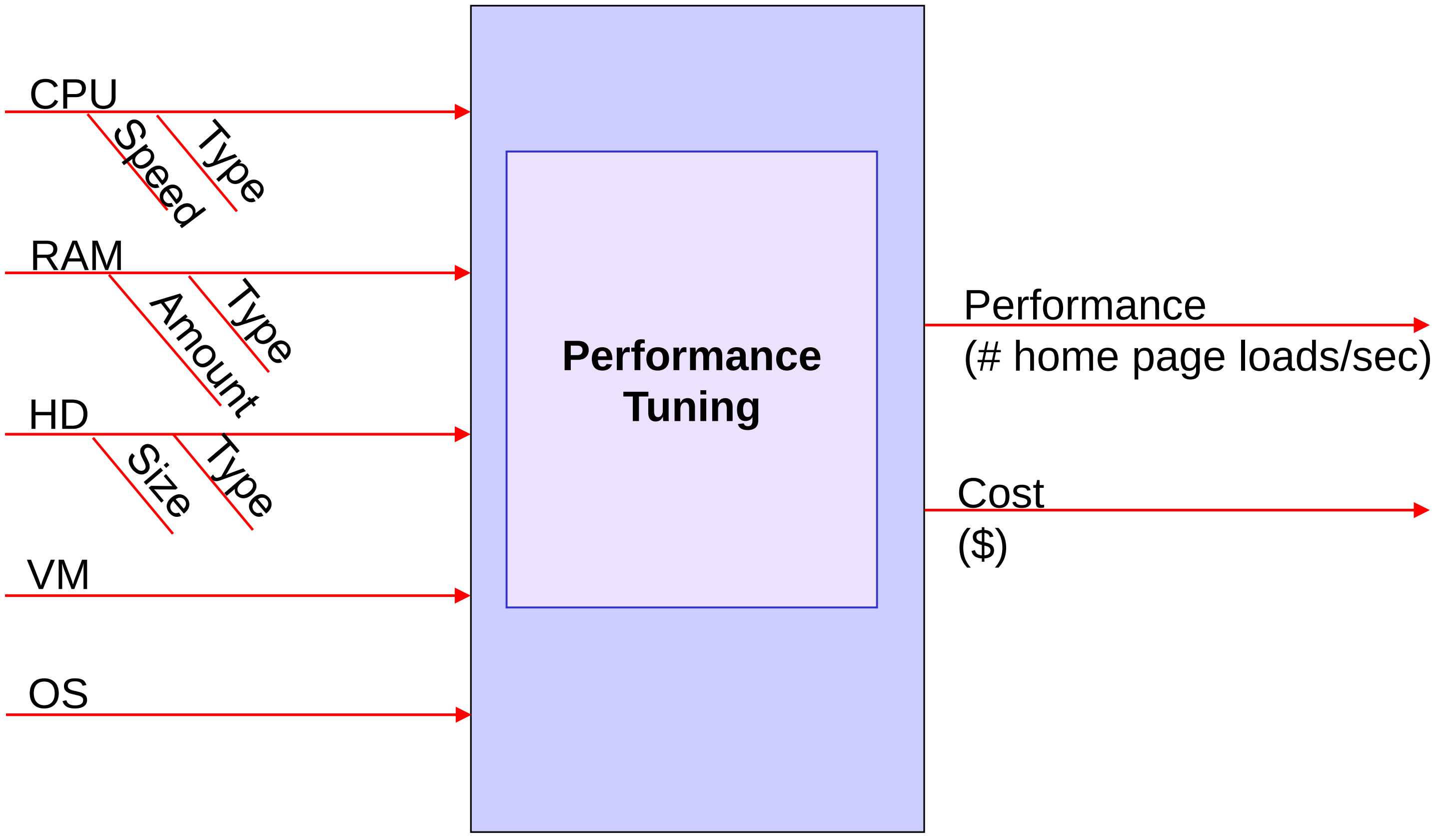
Web-Based Application Process



Combinatorial Test Terminology

Y: Output, response variable, dependent variable

X: Input, factor, independent variable (a measurable entity that is purposely changed during an experiment)

Level: A unique value or choice of a factor (X)

Run: An experimental test combination of the levels of the X's

Replication: Doing or repeating an experimental combination

Effect: The difference or impact on Y when changing X

Interaction: When the effect of one factor depends on the level of another factor

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Performance Tuning Terminology

Factors/Inputs (X's)	Levels (Choices)	Performance/Outputs (Y's)
CPU Type	Itanium, Zeon	# home page loads/sec
CPU Speed	1 GHz, 2.5 GHz	Cost
RAM Amount	2 GB, 4 GB	
HD Size	50 GB, 500 GB	
VM	J2EE, .NET	
OS	Windows, Linux	

Which factors are important? Which are not?
Which combination of factor choices will maximize performance?
How do you know for sure? Show me the data.

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Dilbert on Testing

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Approaches to Testing Multiple Factors

- **Traditional Approaches**
  - One Factor at a Time (OFAT)
  - Oracle (Best Guess)
  - All possible combinations (full factorial)
- **Modern Approach (Scientific Test and Analysis Techniques or STAT)**
  - Statistically designed experiments (DOE) … full factorial plus other orthogonal or nearly orthogonal designs, depending on the situation

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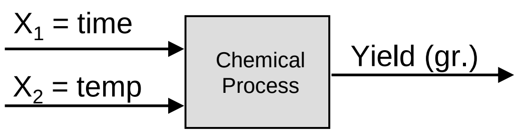
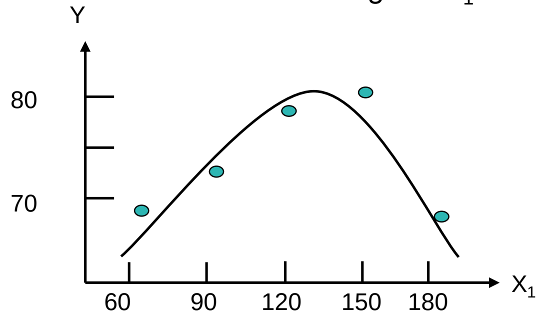
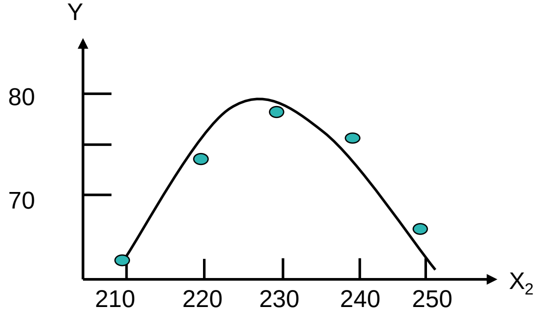
OFAT (One Factor at a Time)



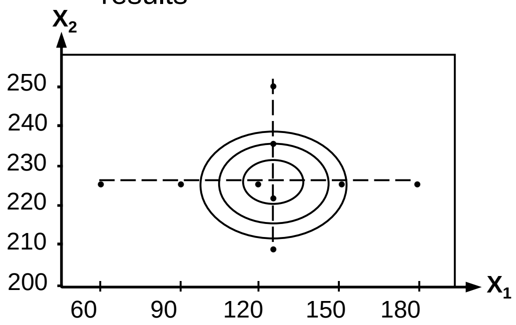
1. Hold X_2 constant and vary X_1. Find the "best setting" for X_1



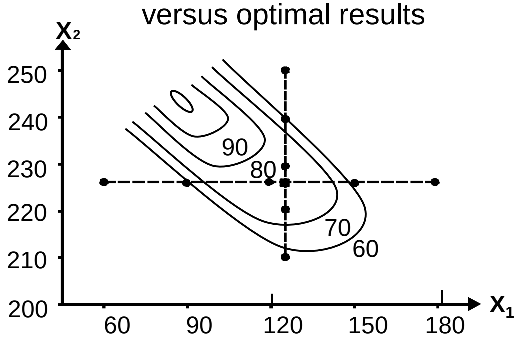
2. Hold X_1 constant at "best setting" and vary X_2. Find the "best setting" for X_2.



3. One factor at a time results



4. One factor at a time results versus optimal results



The Good and Bad about OFAT

- **Good News**
  - Simple
  - Intuitive
  - The way we were originally taught
- **Bad News**
  - Will not be able to estimate variable interaction effects
  - Will not be able to generate prediction models and thus not be able to optimize performance

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Oracle (Best Guess)

A = CPU Type (1=Itanium; 2=Xeon)

B = CPU Speed (1=1 GHz; 2=2.5 GHz)

C = RAM Amount (1=2 GB; 2=4 GB)

D = HD Size (1=50 GB; 2=500 GB)

E = VM (1=J2EE; 2=.NET)

Y = # home page loads/sec

Run	A	B	C	D	E	Y
1	1	2	1	1	1	5
2	1	1	1	1	1	6
3	2	2	1	1	1	5
4	2	1	1	1	2	6
5	1	2	2	2	2	7
6	1	1	2	2	2	8
7	2	2	2	2	2	10
8	2	1	2	2	1	11

Does factor D shift the average of Y?

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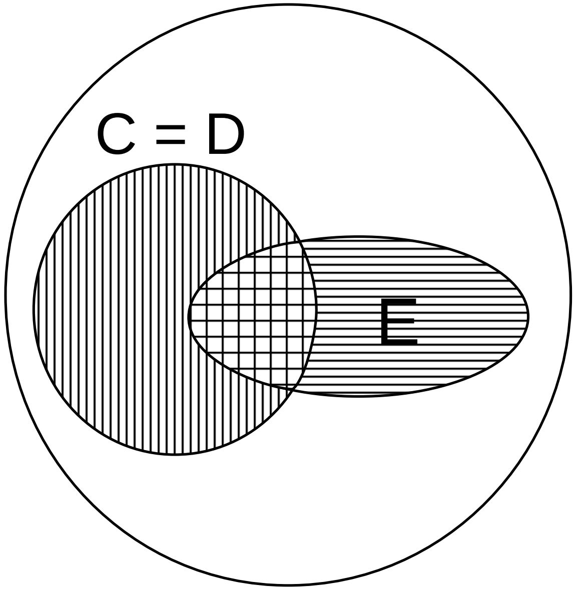
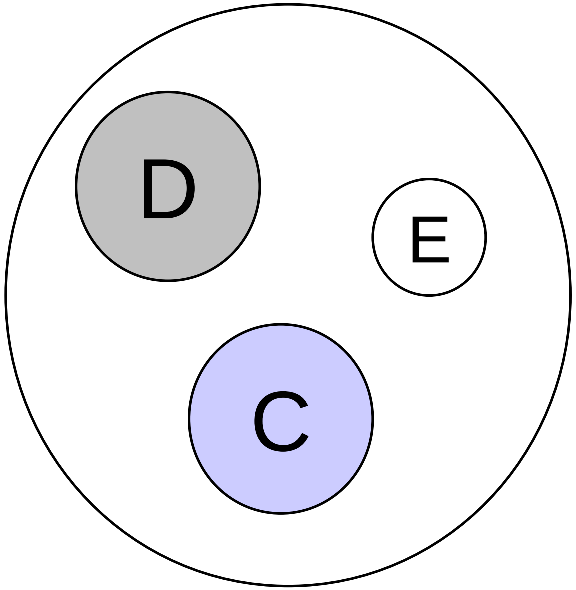
Evaluating the Effects of Variables on Y

What we have is:



What we need is a design to provide independent estimates of effects:



How do we obtain this independence of variables?

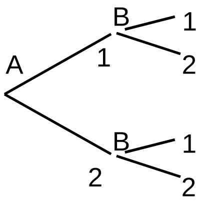
All Possible Combinations (Full Factorial)

MATRIX FORM / **TREE DIAGRAM**

Example 1:

A (2 levels)
B (2 levels)

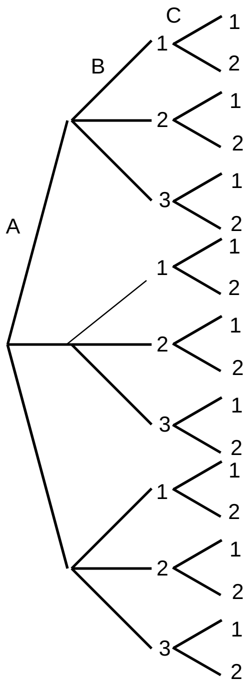
A	B
1	1
1	2
2	1
2	2



Example 2:

A (3 levels)
B (3 levels)
C (2 levels)

A	B	C
1	1	1
1	2	1
1	3	1
2	1	1
2	2	1
2	3	1
3	1	1
3	2	1
3	3	1
1	1	2
1	2	2
1	3	2
2	1	2
2	2	2
2	3	2
3	1	2
3	2	2
3	3	2



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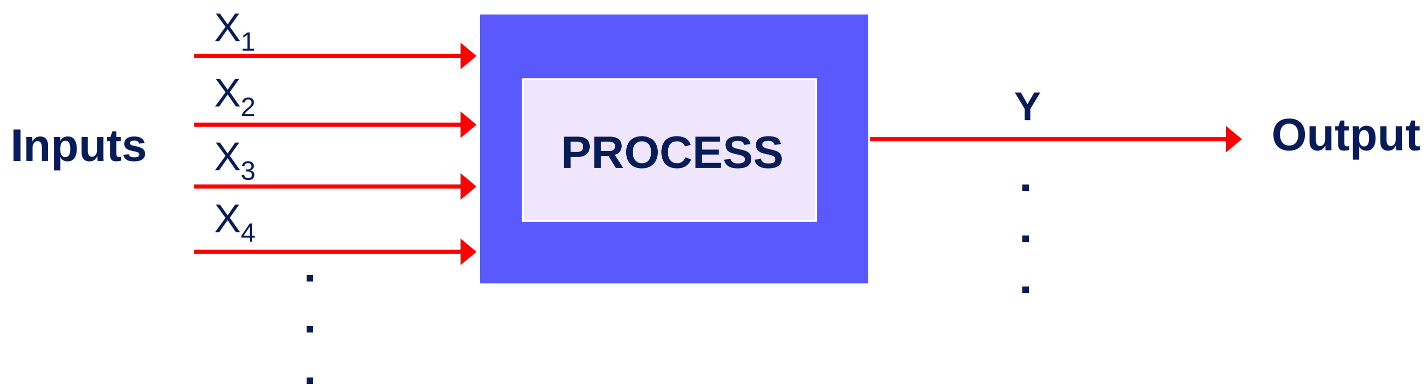
The Purpose of a Designed Experiment

Purposeful changes of the inputs (factors) in order to observe corresponding changes in the output (response).



Run	X_1	X_2	X_3	X_4	Y_1	Y_2	. . . . . .	$\bar{Y}$	S_Y
1									
2									
3									
.									
.									

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Famous Quote

"All experiments (tests) are designed experiments; some are poorly designed, some are well designed."

George Box (1919-2013), Professor of Statistics, DOE Guru

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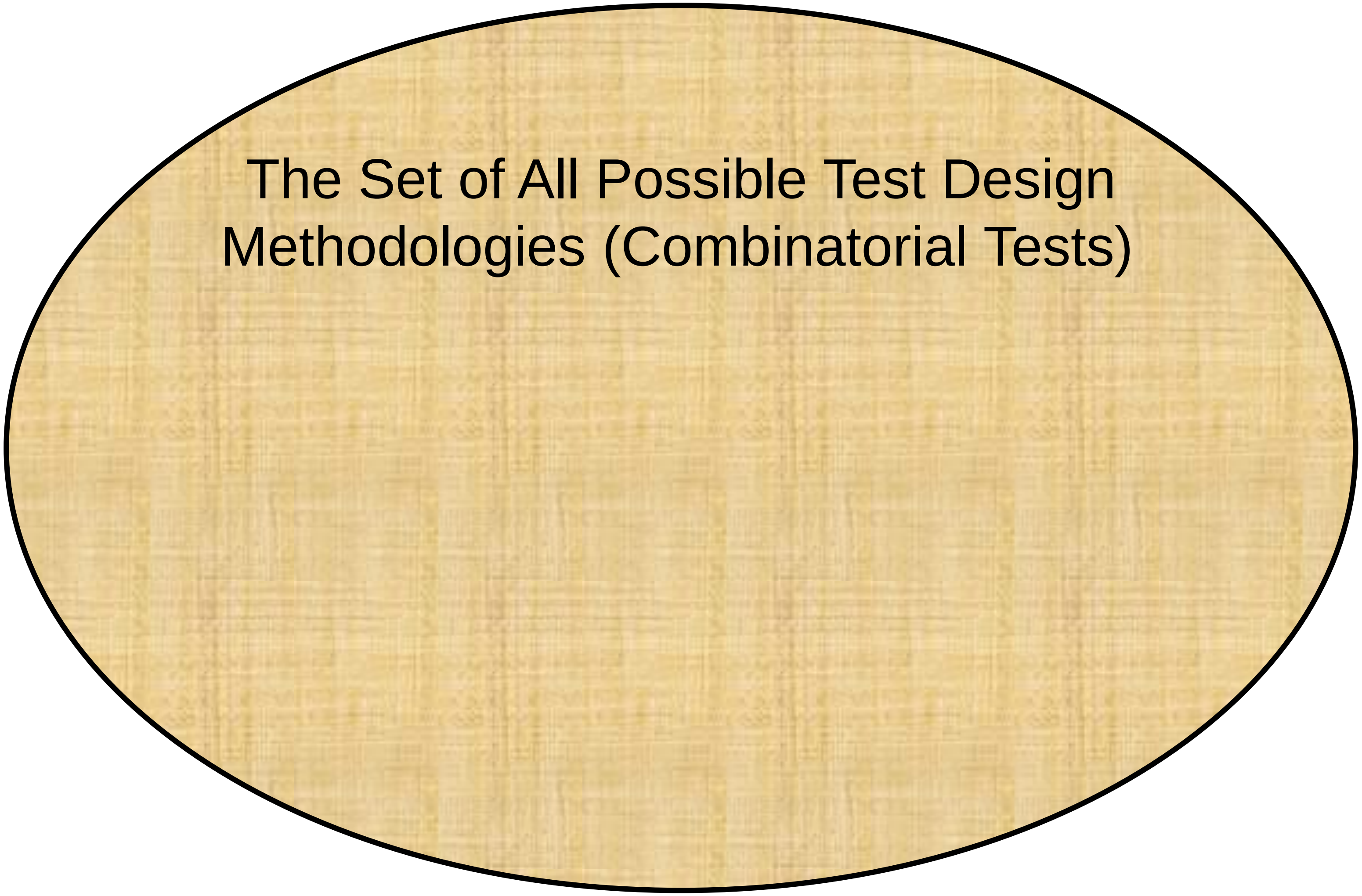
Design of Experiments (DOEs): A Subset of All Possible Test Design Methodologies



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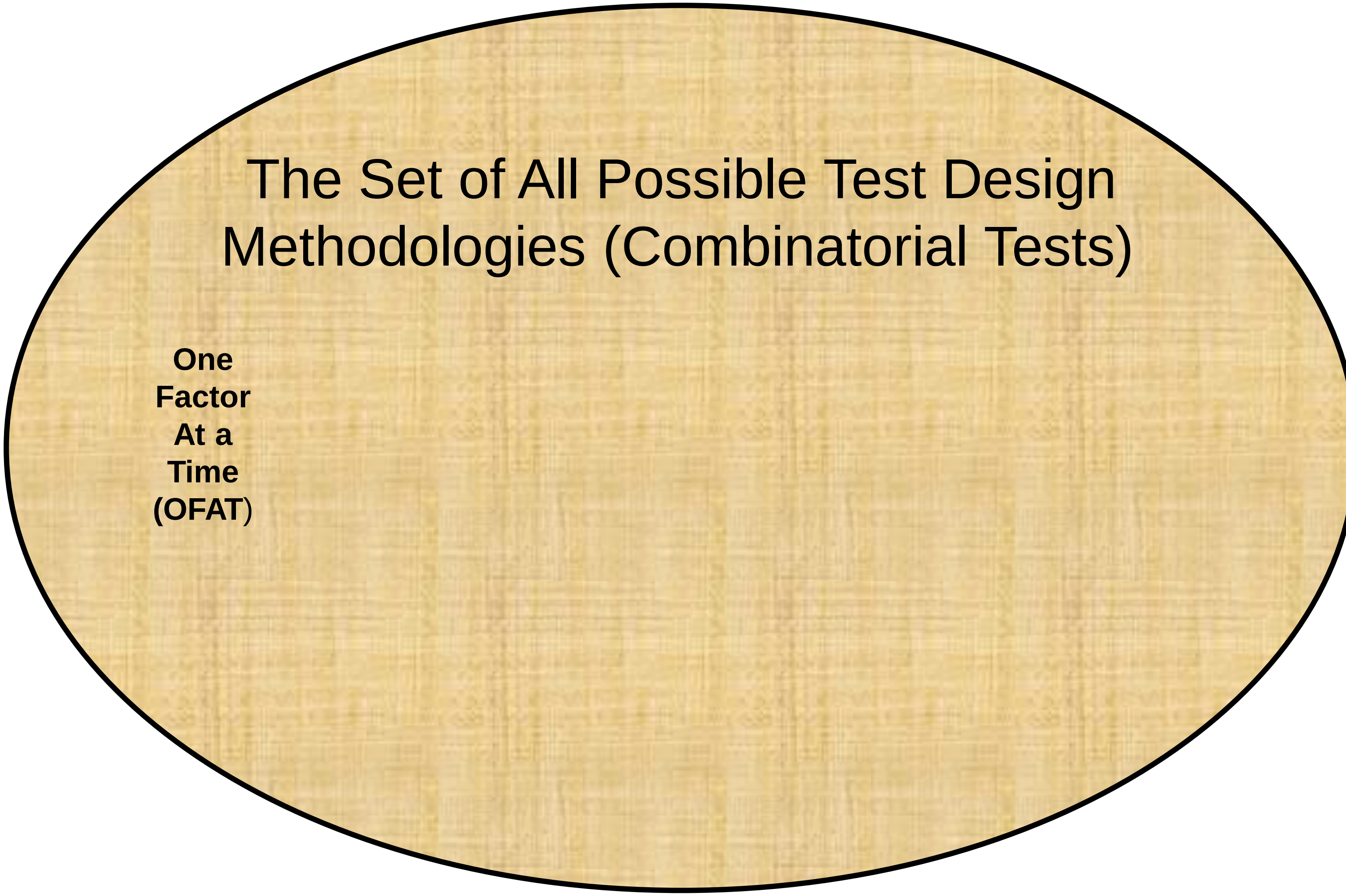

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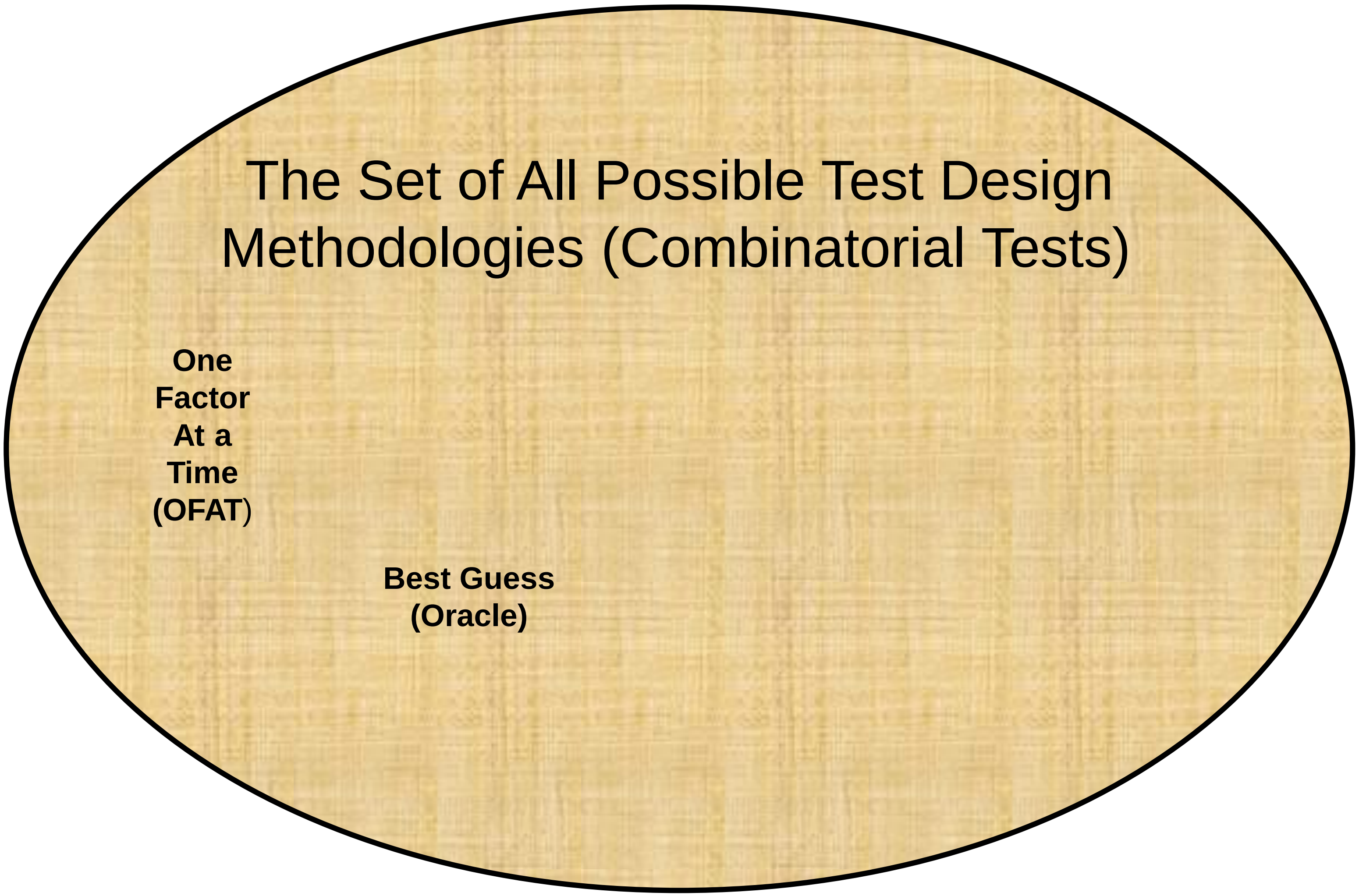

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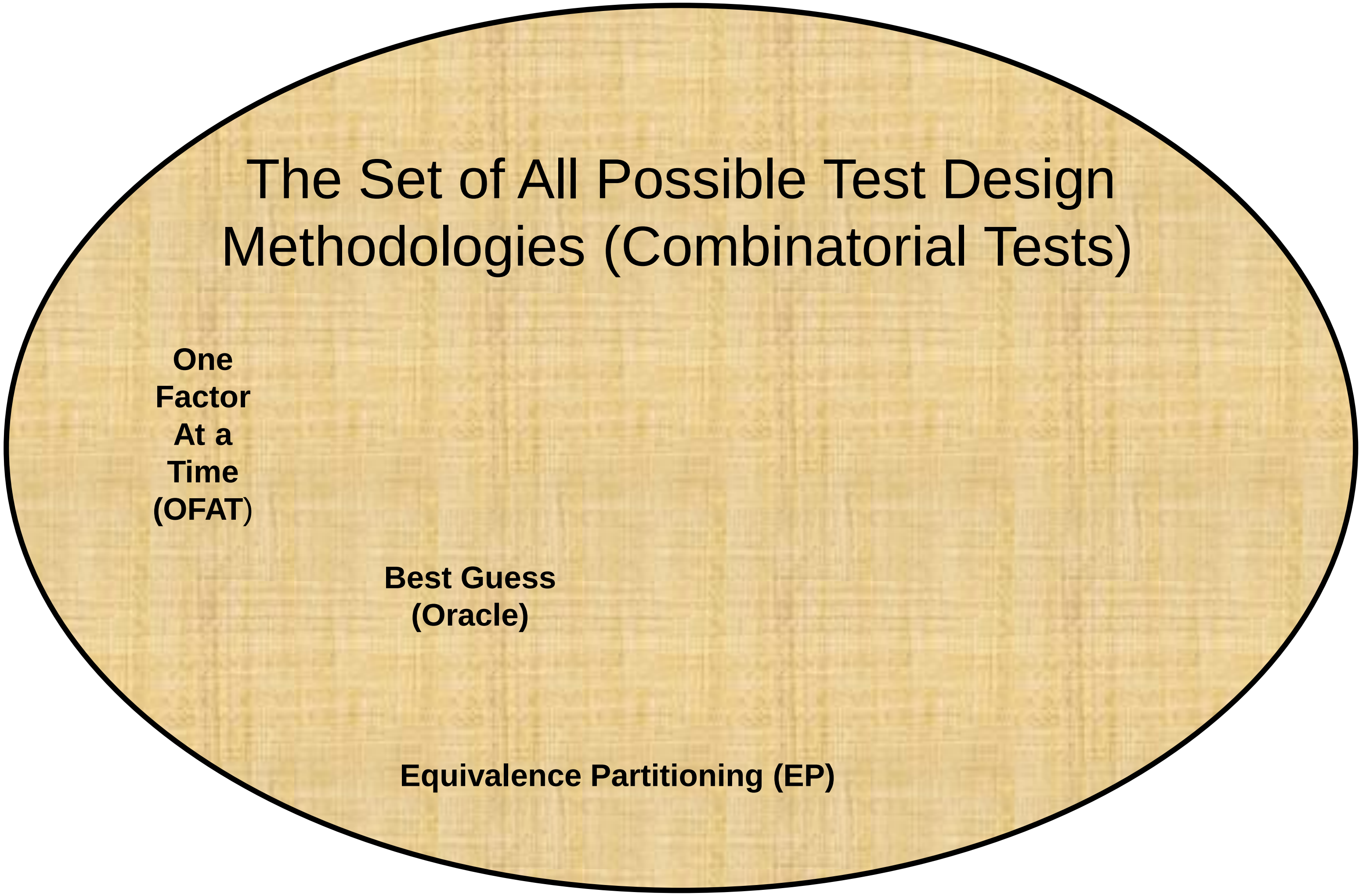

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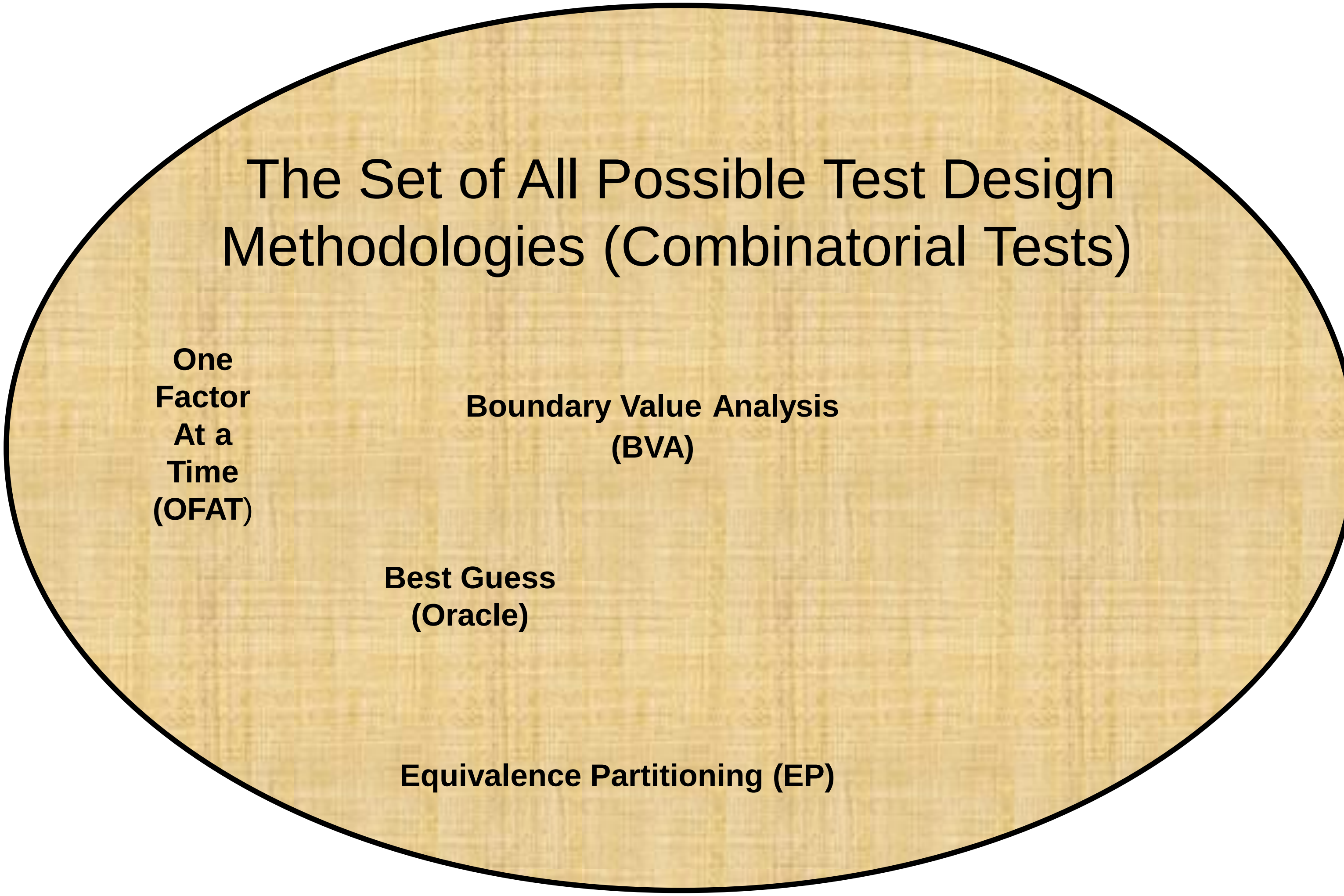

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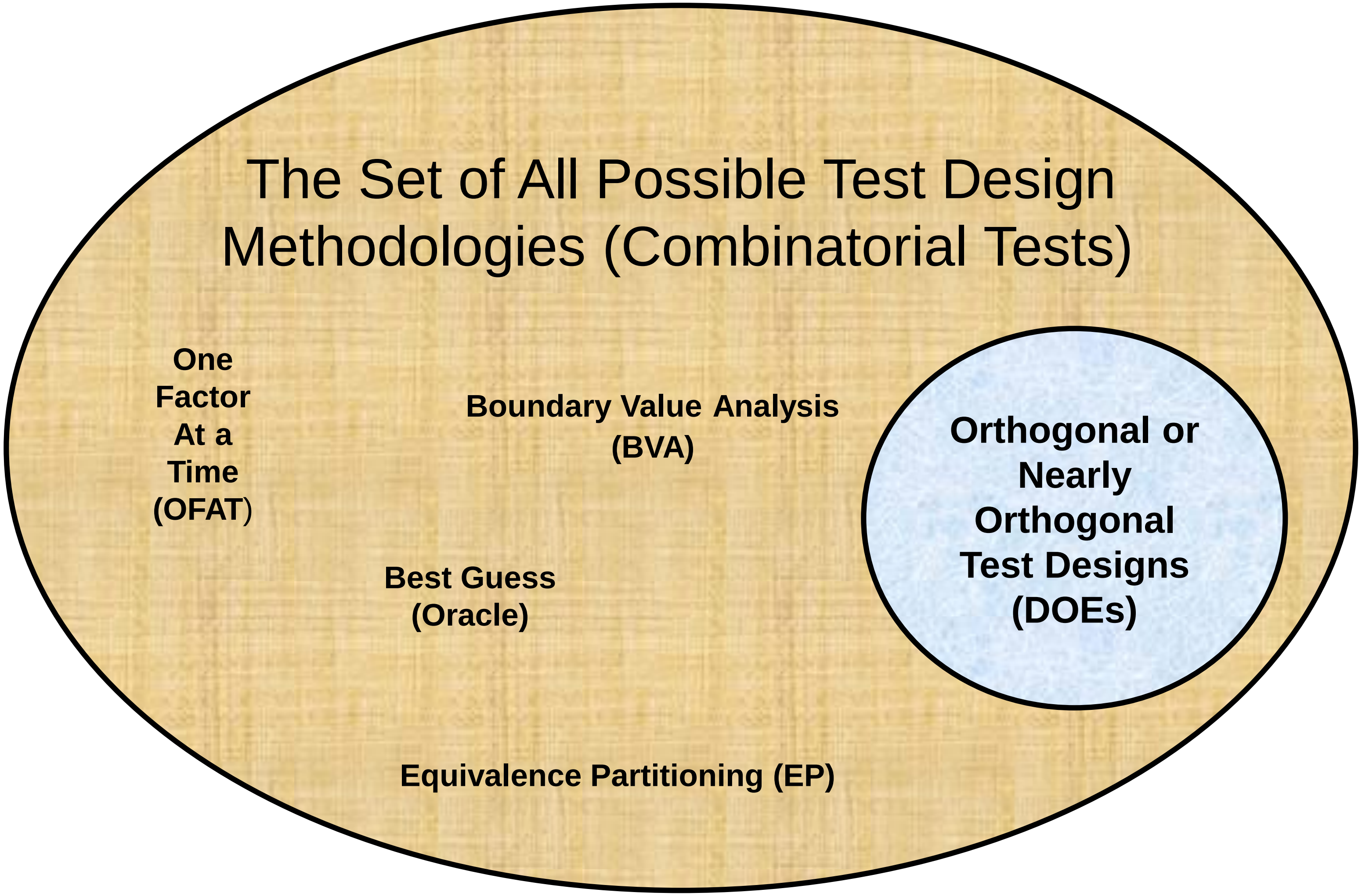

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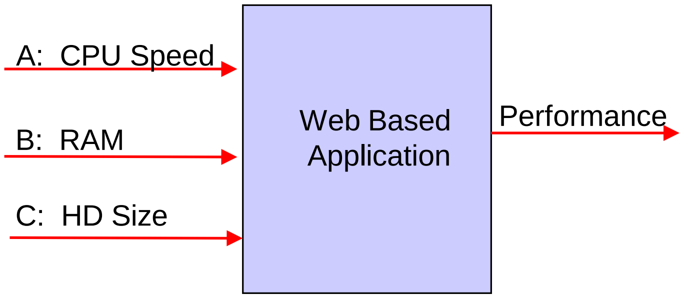
Statistically Designed Experiments (DOE):
Orthogonal or Nearly Orthogonal Designs

- FULL FACTORIALS (for small number of factors)
- FRACTIONAL FACTORIALS
- PLACKETT - BURMAN
- LATIN SQUARES
- HADAMARD MATRICES
- BOX - BEHNKEN DESIGNS
- CENTRAL COMPOSITE DESIGNS
- NEARLY ORTHOGONAL LATIN HYPERCUBE DESIGNS
- HIGH THROUGHPUT TESTING (ALL PAIRS)

(PLACKETT - BURMAN, LATIN SQUARES, HADAMARD MATRICES: Taguchi Designs)

(BOX - BEHNKEN DESIGNS, CENTRAL COMPOSITE DESIGNS: Response Surface Designs)

Example:



SIMPLE DEFINITION OF TWO-LEVEL ORTHOGONAL DESIGNS

Run	Actual Settings (1 GHz, 2.5 GHz) CPU Speed	(2 GB, 4 GB) RAM	(50 GB, 500 GB) HD Size	Coded Matrix (A) CPU Speed	(B) RAM	(C) HD Size	Responses
1							
2							
3							
4							
5							
6							
7							
8							

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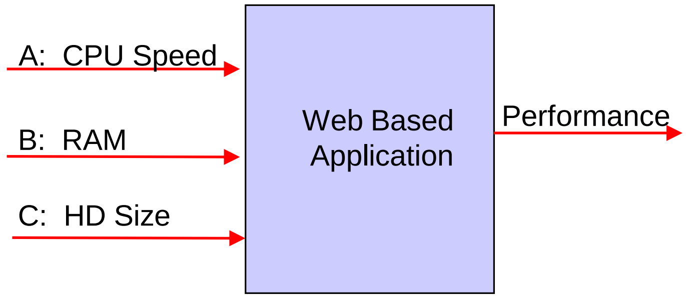

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Run	Actual Settings (1 GHz, 2.5 GHz) CPU Speed	(2 GB, 4 GB) RAM	(50 GB, 500 GB) HD Size	Coded Matrix (A) CPU Speed	(B) RAM	(C) HD Size	Responses
1	1						
2	1						
3	1						
4	1						
5	2.5						
6	2.5						
7	2.5						
8	2.5						

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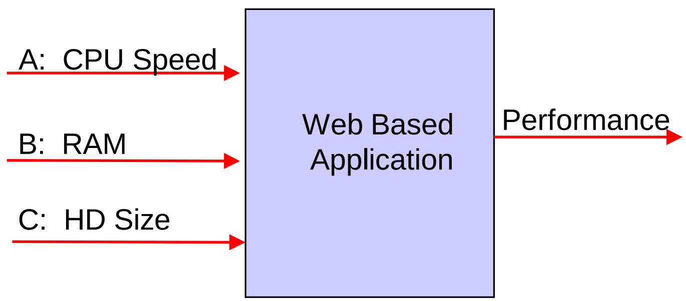

SIMPLE DEFINITION OF TWO-LEVEL ORTHOGONAL DESIGNS

Run	Actual Settings: CPU Speed (1 GHz, 2.5 GHz)	Actual Settings: RAM (2 GB, 4 GB)	Actual Settings: HD Size (50 GB, 500 GB)	Coded Matrix: (A) CPU Speed	Coded Matrix: (B) RAM	Coded Matrix: (C) HD Size	Responses
1	1	2					
2	1	2					
3	1	4					
4	1	4					
5	2.5	2					
6	2.5	2					
7	2.5	4					
8	2.5	4					

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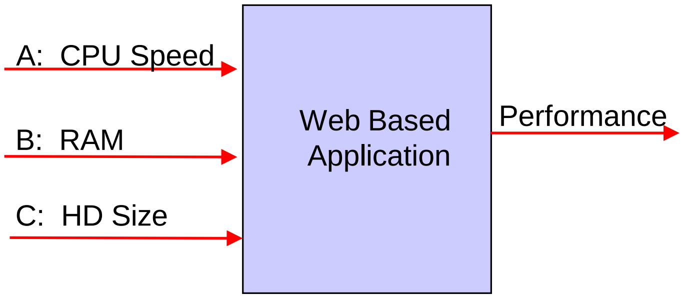

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Run	Actual Settings (1 GHz, 2.5 GHz) CPU Speed	(2 GB, 4 GB) RAM	(50 GB, 500 GB) HD Size	Coded Matrix (A) CPU Speed	(B) RAM	(C) HD Size	Responses
1	1	2	50				
2	1	2	500				
3	1	4	50				
4	1	4	500				
5	2.5	2	50				
6	2.5	2	500				
7	2.5	4	50				
8	2.5	4	500				

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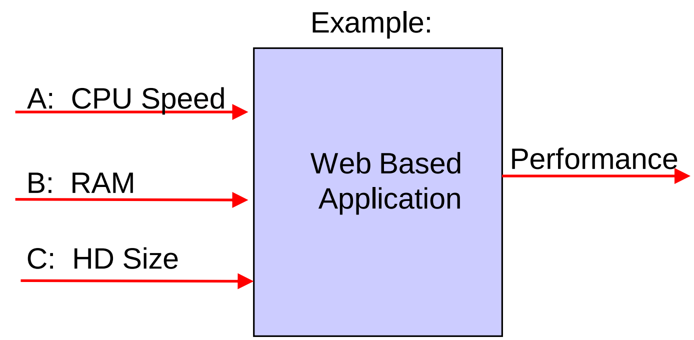

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Run	Actual Settings (1 GHz, 2.5 GHz) CPU Speed	(2 GB, 4 GB) RAM	(50 GB, 500 GB) HD Size	Coded Matrix (A) CPU Speed	(B) RAM	(C) HD Size	Responses
1	1	2	50	-1	-1	-1	
2	1	2	500	-1	-1	+1	
3	1	4	50	-1	+1	-1	
4	1	4	500	-1	+1	+1	
5	2.5	2	50	+1	-1	-1	
6	2.5	2	500	+1	-1	+1	
7	2.5	4	50	+1	+1	-1	
8	2.5	4	500	+1	+1	+1	

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The Beauty of Orthogonality:
independent evaluation of effects

A Full Factorial Design for 3 Factors, Each at 2 Levels

Run	A	B	C
1	-	-	-
2	-	-	+
3	-	+	-
4	-	+	+
5	+	-	-
6	+	-	+
7	+	+	-
8	+	+	+

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The Beauty of Orthogonality: independent evaluation of effects

A Full Factorial Design for 3 Factors, Each at 2 Levels

Run	A	B	C	AB
1	-	-	-	+
2	-	-	+	+
3	-	+	-	-
4	-	+	+	-
5	+	-	-	-
6	+	-	+	-
7	+	+	-	+
8	+	+	+	+

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A Full Factorial Design for 3 Factors, Each at 2 Levels

Run	A	B	C	AB	AC
1	-	-	-	+	+
2	-	-	+	+	-
3	-	+	-	-	+
4	-	+	+	-	-
5	+	-	-	-	-
6	+	-	+	-	+
7	+	+	-	+	-
8	+	+	+	+	+

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independent evaluation of effects

A Full Factorial Design for 3 Factors, Each at 2 Levels

Run	A	B	C	AB	AC	BC
1	-	-	-	+	+	+
2	-	-	+	+	-	-
3	-	+	-	-	+	-
4	-	+	+	-	-	+
5	+	-	-	-	-	+
6	+	-	+	-	+	-
7	+	+	-	+	-	-
8	+	+	+	+	+	+

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The Beauty of Orthogonality:
independent evaluation of effects

A Full Factorial Design for 3 Factors, Each at 2 Levels

Run	A	B	C	AB	AC	BC	ABC
1	-	-	-	+	+	+	-
2	-	-	+	+	-	-	+
3	-	+	-	-	+	-	+
4	-	+	+	-	-	+	-
5	+	-	-	-	-	+	+
6	+	-	+	-	+	-	-
7	+	+	-	+	-	-	-
8	+	+	+	+	+	+	+

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What can DOE do for us?

- An optimal data collection methodology
- "Interrogates" the process
- Used to identify important relationships between inputs and outputs
- Identifies important interactions between process variables
- Can be used to optimize a process and assess risk
- Changes "I think" to "I know"

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Three Major Reasons for Using a DOE

- **Screening**
  - For testing many factors in order to **separate** the critical factors from the trivial many.
- **Modeling**
  - For **building functions** that can be used to predict outcomes, assess risk, and optimize performance. These include the ability to evaluate interaction and higher order effects. This is also called characterizing the performance.
- **Performance Verification and Validation**
  - For **confirming** that a system performs in accordance with its specifications/requirements.

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Key Considerations for Determining the Test Design

- The Purpose of the Test
  (Screening, Modeling, Performance Validation)

- Number of Factors (k)

- Number of Levels each factor is to be tested at

- Number of replications (sample size), which will be dependent upon the desired confidence and power of the test

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Two Types of Risk in Evaluating the Result of a Test

- α Risk = **P(false detection)** means we falsely concluded that a factor is important
  - **p-value** gives the exact **P(false detection)**
  - **Confidence** = [1 – **p-value**] x 100%
  - Rule of Thumb (ROT) for "highly significant" result: Confidence ≥ 95%

- β Risk = **P(missed detection)** means we failed to detect something important
  - **Power** = [1 – **P(missed detection)**] x 100%
  - Rule of Thumb (ROT) for sufficient power: Power ≥ 75%
  - A Priori (prior to the test) power calculations are good for test planning purposes, and sample size is the way we can control the power of the test.

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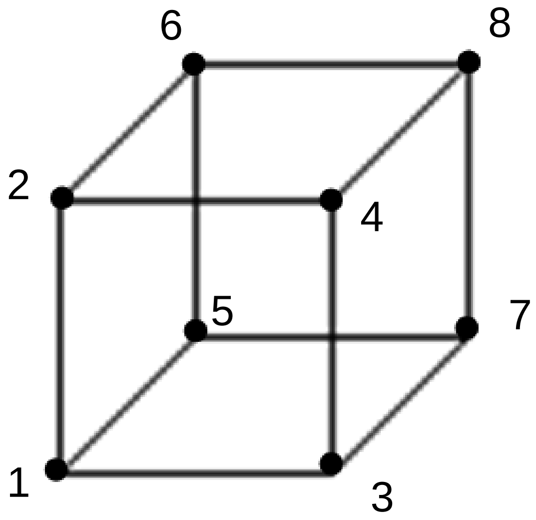
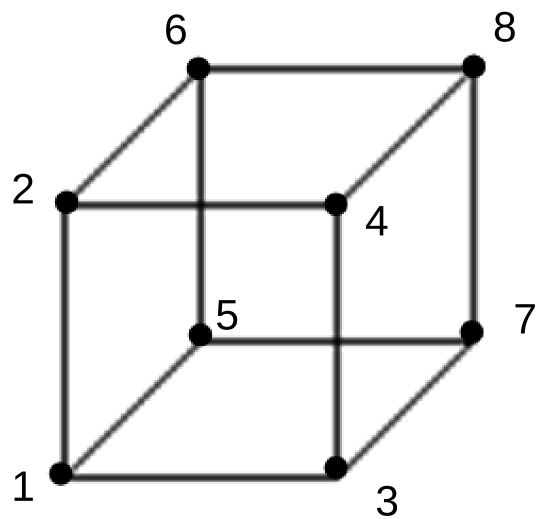
Full Factorial vs. Fractional Factorial
(3 factors at 2 levels)

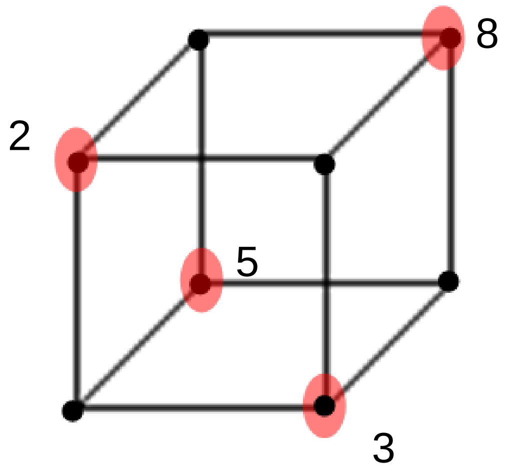


2^3 = 8-run Full Factorial Design

Full Factorial vs. Fractional Factorial
(3 factors at 2 levels)

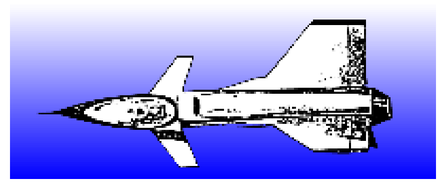


2^3 = 8-run Full Factorial Design



2^{3-1} = 4-run Fractional Factorial Design

Value Delivery: Reducing Time to Market for New Technologies



INPUT

OUTPUT

Pitch <) (0, 15, 30)

Roll <) (0, 15, 30)

W1F <) (-15, 0, 15)

W2F <) (-15, 0, 15)

W3F <) (-15, 0, 15)

Modeling Flight Characteristics of New 3-Wing Aircraft

Six Aero-Characteristics

- **Total # of Combinations = $3^5 = 243$**
- **Central Composite Design: n = 30**

Patent Holder: Dr. Bert Silich

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Aircraft Equations

C_L = .233 + .008(P)^2 + .255(P) + .012(R) - .043(WD1) - .117(WD2) + .185(WD3) + .010(P)(WD3) - .042(R)(WD1) + .035(R)(WD2) + .016(R)(WD3) + .010(P)(R) - .003(WD1)(WD2) - .006(WD1)(WD3)

C_D = .058 + .016(P)^2 + .028(P) - .004(WD1) - .013(WD2) + .013(WD3) + .002(P)(R) - .004(P)(WD1) - .009(P)(WD2) + .016(P)(WD3) - .004(R)(WD1) + .003(R)(WD2) + .020(WD1)^2 + .017(WD2)^2 + .021(WD3)^2

C_Y = -.006(P) - .006(R) + .169(WD1) - .121(WD2) - .063(WD3) - .004(P)(R) + .008(P)(WD1) - .006(P)(WD2) - .008(P)(WD3) - .012(R)(WD1) - .029(R)(WD2) + .048(R)(WD3) - .008(WD1)^2

C_M = .023 - .008(P)^2 + .004(P) - .007(R) + .024(WD1) + .066(WD2) - .099(WD3) - .006(P)(R) + .002(P)(WD2) - .005(P)(WD3) + .023(R)(WD1) - .019(R)(WD2) - .007(R)(WD3) + .007(WD1)^2 - .008(WD2)^2 + .002(WD1)(WD2) + .002(WD1)(WD3)

C_{YM}= .001(P) + .001(R) - .050(WD1) + .029(WD2) + .012(WD3) + .001(P)(R) - .005(P)(WD1) - .004(P)(WD2) - .004(P)(WD3) + .003(R)(WD1) + .008(R)(WD2) - .013(R)(WD3) + .004(WD1)^2 + .003(WD2)^2 - .005(WD3)^2

C_e = .003(P) + .035(WD1) + .048(WD2) + .051(WD3) - .003(R)(WD3) + .003(P)(R) - .005(P)(WD1) + .005(P)(WD2) + .006(P)(WD3) + .002(R)(WD1)

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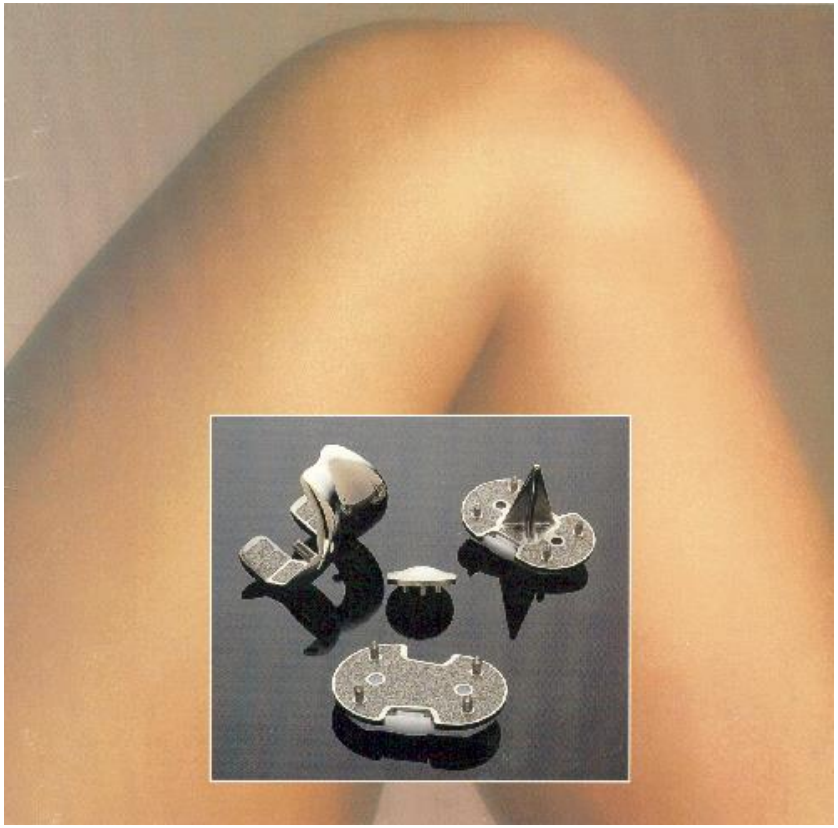
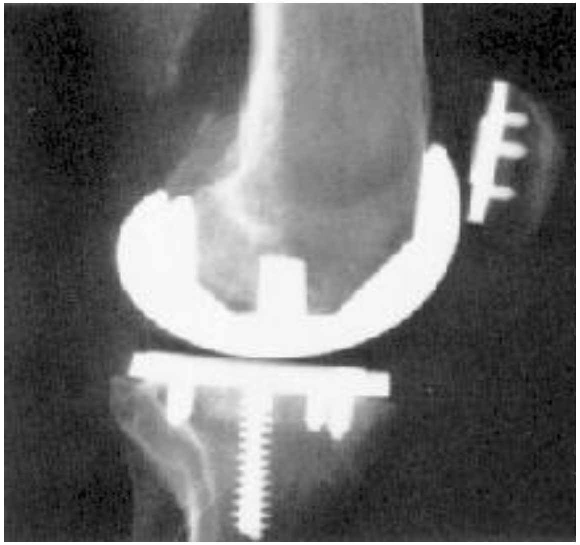
Fusing Titanium and Cobalt-Chrome





Courtesy Rai Chowdhary

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DOE "Market Research" Example

Suppose that, in the auto industry, we would like to investigate the following automobile attributes (i.e., factors), along with accompanying levels of those attributes:

A: Brand of Auto:	-1 = foreign		+1 = domestic
B: Auto Color:	-1 = light	0 = bright	+1 = dark
C: Body Style:	-1 = 2-door	0 = 4-door	+1 = sliding door/hatchback
D: Drive Mechanism:	-1 = rear wheel	0 = front wheel	+1 = 4-wheel
E: Engine Size:	-1 = 4-cylinder	0 = 6-cylinder	+1 = 8-cylinder
F: Interior Size:	-1 ≤ 2 people	0 = 3-5 people	+1 ≥ 6 people
G: Gas Mileage:	-1 ≤ 20 mpg	0 = 20-30 mpg	+1 ≥ 30 mpg
H: Price:	-1 ≤ $20K	0 = $20-$40K	+1 ≥ $40K

In addition, suppose the respondents chosen to provide their preferences to product profiles are taken based on the following demographic:

J: Age:	-1 ≤ 25 years old	+1 ≥ 35 years old
K: Income:	-1 ≤ $30K	+1 ≥ $40K
L: Education:	-1 < BS	+1 ≥ BS

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DOE "Market Research" Example (cont.)

Question: Choose the best design for evaluating this scenario

Answer: L_{18} design with attributes A - H in the inner array and factors J, K, and L in the outer array, resembling an L_{18} robust design, as shown below:

	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8
L	-	+	-	+	-	+	-	+
K	-	-	+	+	-	-	+	+
J	-	-	-	-	+	+	+	+

Run*	A	B	C	D	E	F	G	H	y_1 … y_8	$\bar{y}$	s
1	-	-	-	-	-	-	-	-	Segmentation of the population or <u>Respondent Profiles</u>		
2	-	-	0	0	0	0	0	0			
3	-	-	+	+	+	+	+	+			
4	-	0	-	-	0	0	+	+			
5	-	0	0	0	+	+	-	-			
6	-	0	+	+	-	-	0	0			
7	-	+	-	0	-	+	0	+			
8	-	+	0	+	0	-	+	-			
9	-	+	+	-	+	0	-	0			
10	+	-	-	+	+	0	0	-			
11	+	-	0	-	-	+	+	0			
12	+	-	+	0	0	-	-	+			
13	+	0	-	0	+	-	+	0			
14	+	0	0	+	-	0	-	+			
15	+	0	+	-	0	+	0	-			
16	+	+	-	+	0	+	-	0			
17	+	+	0	-	+	-	0	+			
18	+	+	+	0	-	0	+	-			

* 18 different product profiles

Google on DOE
(quotes* from Daryl Pregibon, Google Engineer)

"From a user's perspective, a query was submitted and results appear. From Google's perspective, the user has provided an opportunity to test something. What can we test? Well, there is so much to test that we have an Experiment Council that vets experiment proposals and quickly approves those that pass muster."

" We evangelize experimentation to the extent that we provide a mechanism for advertisers to run their own experiments.

. . . allows an advertiser to run a (full) factorial experiment on its web page. Advertisers can explore layout and content alternatives while Google randomly directs queries to the resulting treatment combinations. Simple analysis of click and conversion rates allows advertisers to explore a range of alternatives and their effect on user awareness and interest."

*** Taken From: *Statistics @ Google* in Amstat News, May 2011**

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Growth Rate of Full-Factorial Designs

For 2-level designs and k factors: 2^k combinations

- for k = 2 factors: 2^2 = 4 combinations
- for k = 3 factors: 2^3 = 8 combinations
- for k = 10 factors: 2^{10} = 1,024 combinations

For 3-level designs and k factors: 3^k combinations

- for k = 2 factors: 3^2 = 9 combinations
- for k = 3 factors: 3^3 = 27 combinations
- for k = 10 factors: 3^{10} = 59,049 combinations

What if the # of factors and/or the number of levels gets large?

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Latin Hypercube Designs
(space filling designs)

- Method to populate the design space when using deterministic simulation models or when many variables are involved.

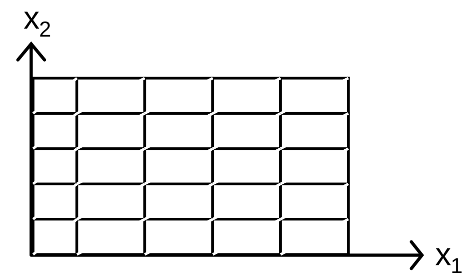
- Design space has k variables (or dimensions).

  Ex: Assume k = 2

- Suppose a sample of size n is to be taken; Stratify the design space into n^k cells.

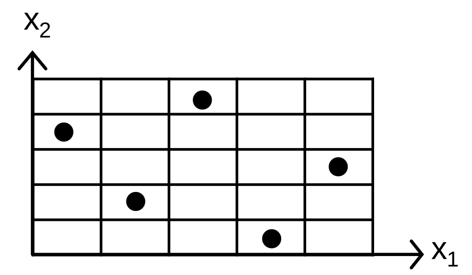
  Ex: Assume n = 5; $n^k = 5^2 = 25$

  Note: there are n=5 strata for each of the k=2 dimensions.



- Each of the n points is sampled such that each marginal strata is represented only once in the sample.

  Note: each sample point has its own unique row and column.



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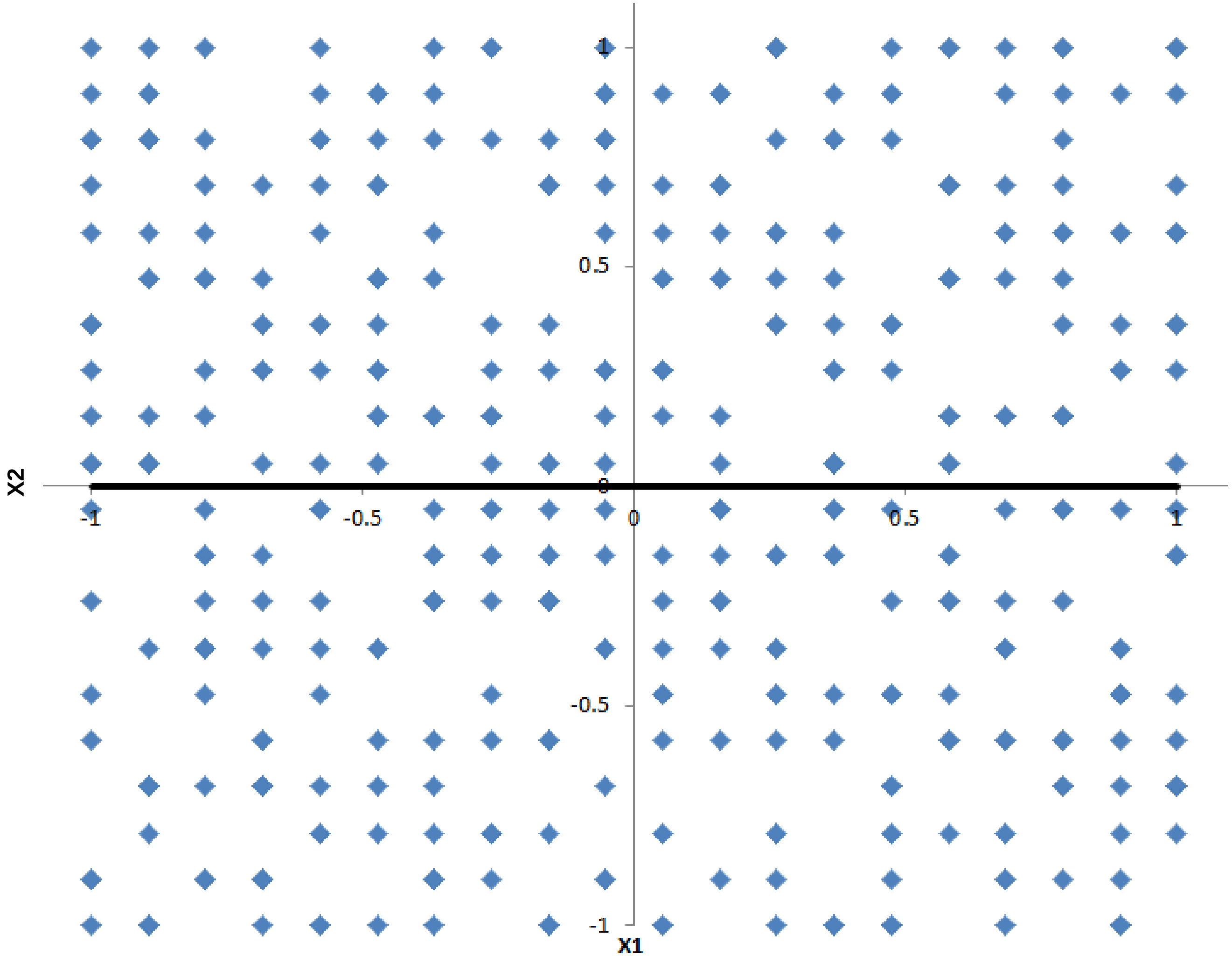
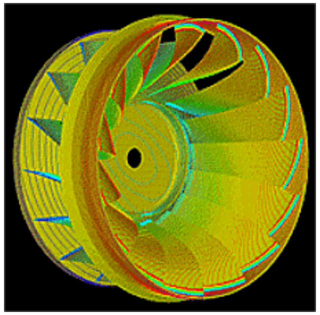
Nearly Orthogonal Latin Hypercube Design
(20 variables each at 20 levels projected onto x1 vs x2)



Note the balance in the design.

Examples of Simulation and High Performance Computing (HPC)

Power

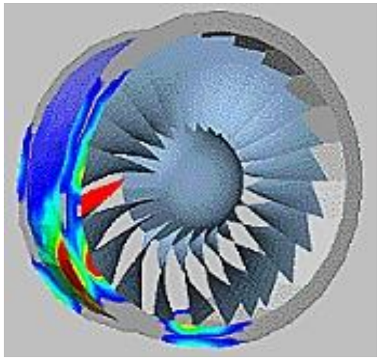


Simulation of stress and vibrations of turbine assembly for use in nuclear power generation

Automotive

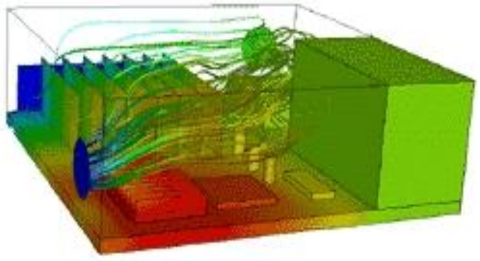
Simulation of underhood thermal cooling for decrease in engine space and increase in cabin space and comfort

Aerospace



Evaluation of dual bird-strike on aircraft engine nacelle for turbine blade containment studies

Electronics



Evaluation of cooling air flow behavior inside a computer system chassis

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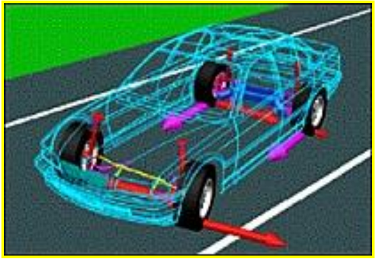
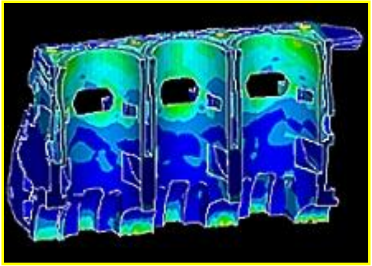
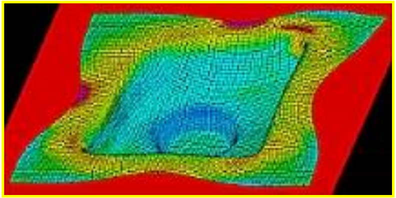
Examples of Computer Aided Engineering (CAE) and Simulation Software



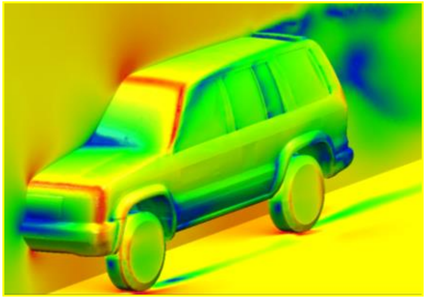
Mechanical motion: **Multibody kinetics and dynamics**
- ADAMS®
- DADS



Implicit Finite Element Analysis: **Linear and nonlinear statics, dynamic response**
- MSC.Nastran™, MSC.Marc™
- ANSYS®
- Pro MECHANICA
- ABAQUS® Standard and Explicit
- ADINA



Explicit Finite Element Analysis : **Impact simulation, metal forming**
- LS-DYNA
- RADIOSS
- PAM-CRASH®, PAM-STAMP



General Computational Fluid Dynamics: **Internal and external flow simulation**
- STAR-CD
- CFX-4, CFX-5
- FLUENT®, FIDAP™
- PowerFLOW®

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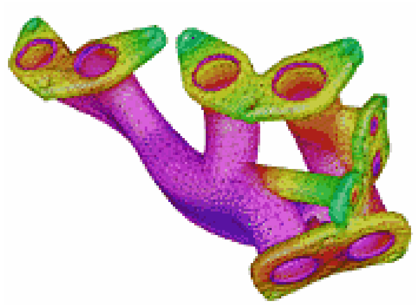
Examples of Computer Aided Engineering (CAE) and Simulation Software (cont.)

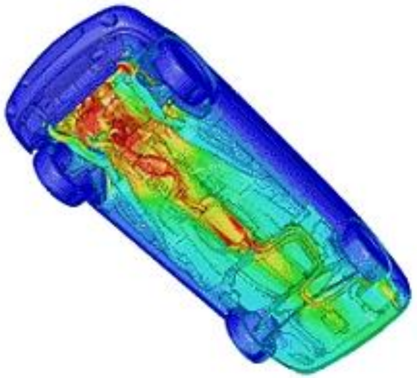
Preprocessing: Finite Element Analysis and Computational Fluid Dynamics mesh generation

- ICEM-CFD
- Gridgen
- Altair® HyperMesh®
- I-deas®
- MSC.Patran
- TrueGrid®
- GridPro
- FEMB
- ANSA



Postprocessing: Finite Element Analysis and Computational Fluid Dynamics results visualization

- Altair® HyperMesh®
- I-deas
- MSC.Patran
- FEMB
- EnSight
- FIELDVIEW
- ICEM CFD Visual3 2.0 (PVS)
- COVISE



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Applying Modeling and Simulation to Automotive Vehicle Design

IDENTIFY CTCs, CDPs → Many, Many x's → **SCREENING DESIGN (NOLHD)** → The critical few CDP's

Examples of CTCs:

- y_1 = weight of vehicle
- y_2 = cost of vehicle

Safety CTCs with constraints specified by FMVSS (Federal Motor Vehicle Safety Standards) — RADIOSS, DYNA, MADYMO:

- y_3 = frontal head impact
- y_4 = frontal chest impact
- y_5 = toe board intrusion
- y_6 = hip deflection
- y_7 = rollover impact
- y_8 = side impact

no federal requirements on these CTCs:

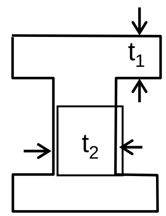
- y_9 = internal aerodynamics (airflow) — CFD
- y_{10} = external aerodynamics (airflow) — CFD
- y_{11} = noise — NASTRAN
- y_{12} = vibration (e.g., steering wheel) — NASTRAN
- y_{13} = harshness (e.g., over bumps, shocks) — NASTRAN
- y_{14} = durability (at 100K miles) — NASTRAN

NASTRAN RADIOSS MADYMO

Integrated processes with high fidelity CAE analyses on HPC servers

Examples of Critical Design Parameters (CDPs or Xs):

- x_1 = roof panel material
- x_2 = roof panel thickness
- x_3 = door pillar dimensions ⇒ i beam
- x_4 = shape/geometry
- x_5 = windshield glass
- x_6 = hood material, sizing and thickness
- x_7 = under hood panel material, sizing and thickness



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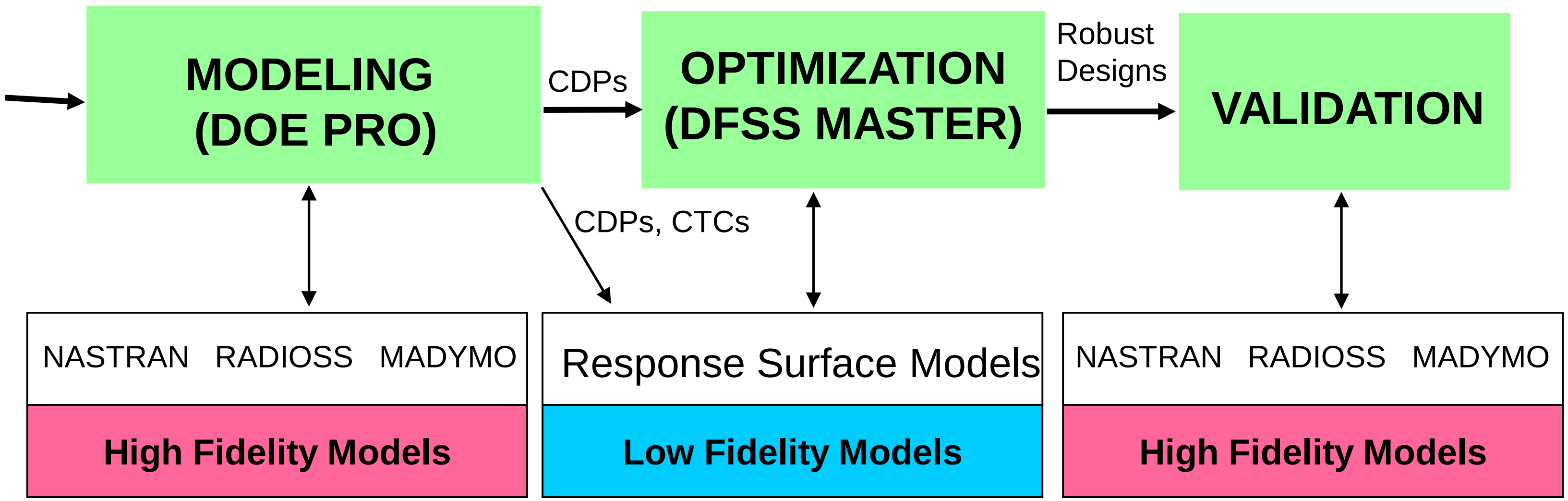
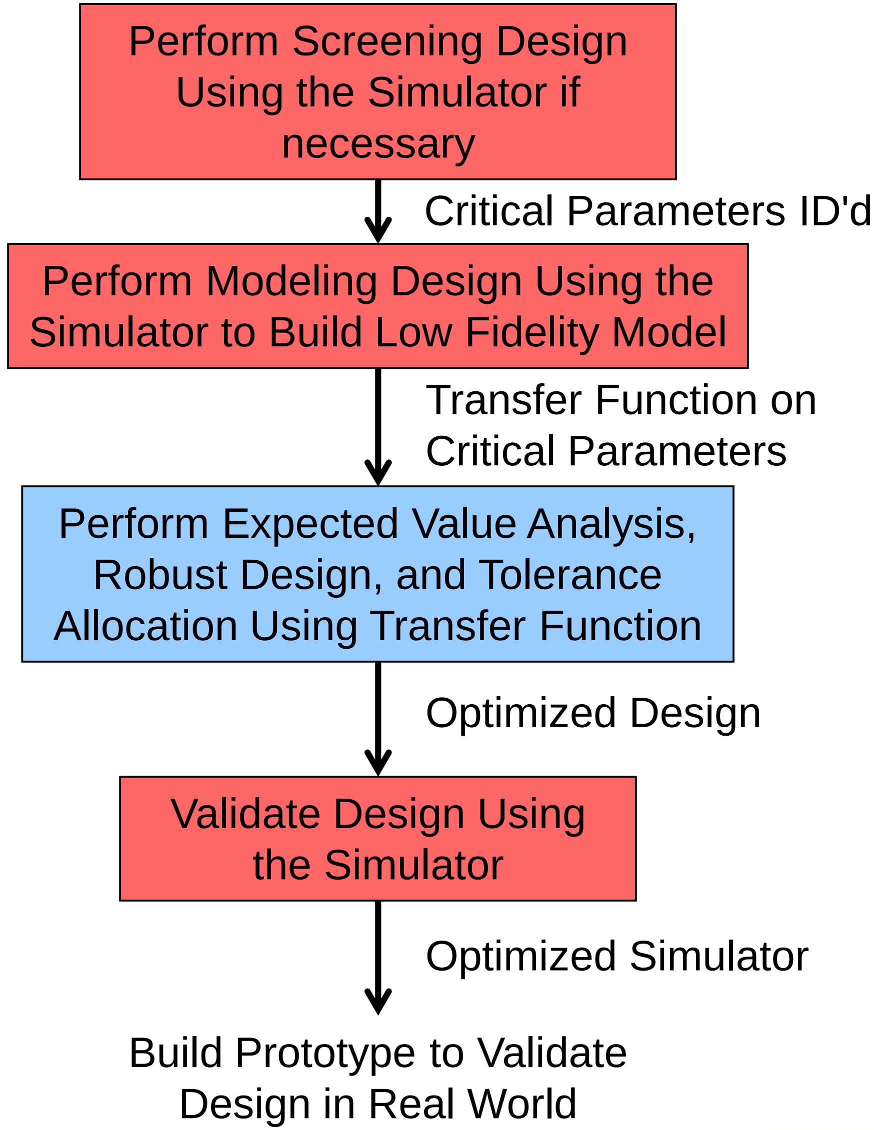
Applying Modeling and Simulation to Automotive Vehicle Design (cont.)



Using DOE to "Optimize the Simulator"



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Environments Where Simulation and Modeling Is Beneficial

- **A high number of design variables**
- **A substantial number of design subsystems and engineering disciplines**
- **Interdependency and interaction between the subsystems and variables**
- **Multiple response variables**
- **Need to characterize the system at a higher level of abstraction**
- **Time and/or space must be compressed**

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Introduction to High Throughput Testing (HTT)

- A recently developed technique based on combinatorics
- Used to test myriad combinations of many factors (typically qualitative) where the factors could have many levels
- Uses a minimum number of runs or test combinations to do this
- Software is needed to select the minimal subset of all possible combinations to be tested so that all 2-way combinations are tested.
- A run or row in an HTT matrix is, like DOE, a combination of different factor levels
- HTT has its origins in the pharmaceutical business where in drug discovery many chemical compounds are combined together at many different strengths to try to produce a reaction.
- Other industries are now using HTT, e.g., software testing, materials discovery, integration and validation testing (see example on next page).

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Basic Combinatorics Relationship

${}_{n}C_{k}$, read as the combination of n things taken k at a time,

$$= \frac{n!}{k!\,(n-k)!}$$

Thus, ${}_{n}C_{2} = n(n-1)/(2\cdot1) = O(n^2)$

and ${}_{n}C_{3} = n(n-1)(n-2)/(3\cdot2\cdot1) = O(n^3)$

etc.

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HTT Example
(Performance Verification and Validation)

- We would like to perform verification testing with the 4 input factors described below.
- All possible combinations would involve how many test combinations?
- If we were interested in testing all pairs only, how many runs would be in the test matrix and what would those combinations be? To answer this question, we used the ProTest software. See next page.

Sensor Type	Weapon Type	External Data Link	Target Type
S1	W1	Yes	T1
S2	W2	No	T2
S3	W3		T3
S4			T4
			T5

High Throughput Testing Example (cont.)
20 Test Cases

	Sensor	Weapon	Data Link	Target
Case 1	S1	W2	Yes	T1
Case 2	S4	W1	Yes	T2
Case 3	S2	W1	No	T3
Case 4	S3	W3	Yes	T4
Case 5	S2	W3	Yes	T5
Case 6	S4	W3	No	T1
Case 7	S3	W2	No	T2
Case 8	S1	W3	Yes	T3
Case 9	S1	W1	No	T4
Case 10	S3	W1	No	T5
Case 11	S2	W1	No	T1
Case 12	S1	W3	No	T2
Case 13	S4	W2	No	T3
Case 14	S2	W2	Yes	T4
Case 15	S4	W2	No	T5
Case 16	S3	W2	Yes	T3
Case 17	S1	W1	Yes	T5
Case 18	S2	W2	Yes	T2
Case 19	S3	W3	Yes	T1
Case 20	S4	W2	No	T4

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High Throughput Testing Example (cont.) Locating the Problem

- If Case 20 were the only failed test, what could be the reason? S4/W2, S4/No, S4/T4, W2/No, W2/T4, No/T4

	Sensor	Weapon	Data Link	Target
Case 1	S1	W2	Yes	T1
Case 2	S4	W1	Yes	T2
Case 3	S2	W1	No	T3
Case 4	S3	W3	Yes	T4
Case 5	S2	W3	Yes	T5
Case 6	S4	W3	No	T1
Case 7	S3	*W2*	*No*	T2
Case 8	S1	W3	Yes	T3
Case 9	S1	W1	*No*	*T4*
Case 10	S3	W1	No	T5
Case 11	S2	W1	No	T1
Case 12	S1	W3	No	T2
Case 13	*S4*	W2	*No*	T3
Case 14	S2	*W2*	Yes	*T4*
Case 15	*S4*	*W2*	No	T5
Case 16	S3	W2	Yes	T3
Case 17	S1	W1	Yes	T5
Case 18	S2	W2	Yes	T2
Case 19	S3	W3	Yes	T1
Case 20	**S4**	**W2**	**No**	**T4**

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High Throughput Testing Example (cont.) Locating the Problem

- If Case 1 were the only failed test, what could be the reason?
  S1/W2, S1/Yes, S1/T1, W2/Yes, W2/T1, Yes/T1

	Sensor	Weapon	Data Link	Target
Case 1	**S1**	**W2**	**Yes**	**T1**
Case 2	S4	W1	Yes	T2
Case 3	S2	W1	No	T3
Case 4	S3	W3	Yes	T4
Case 5	S2	W3	Yes	T5
Case 6	S4	W3	No	T1
Case 7	S3	W2	No	T2
Case 8	S1	W3	Yes	T3
Case 9	S1	W1	No	T4
Case 10	S3	W1	No	T5
Case 11	S2	W1	No	T1
Case 12	S1	W3	No	T2
Case 13	S4	W2	No	T3
Case 14	S2	***W2***	***Yes***	T4
Case 15	S4	W2	No	T5
Case 16	S3	W2	Yes	T3
Case 17	***S1***	W1	***Yes***	T5
Case 18	S2	W2	Yes	T2
Case 19	S3	W3	***Yes***	***T1***
Case 20	S4	W2	No	T4

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Submarine Threat Detection Example

Suppose we want to perform a verification test with the following 7 input factors (with their respective settings):

- Submarine Type (S1, S2, S3)
- Ocean Depth (Shallow, Deep, Very Deep)
- Sonar Type (Active, Passive)
- Target Depth (Surface, Shallow, Deep, Very Deep)
- Sea Bottom (Rock, Sand, Mud)
- Control Mode (Autonomous, Manual)
- Ocean Current (Strong, Moderate, Minimal)

All possible combinations would involve how many runs in the test?

(3x3x2x4x3x2x3 = 1296)

If we were interested in testing all pairs only, how many runs would be in the test? Pro Test generated the following test matrix.

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Submarine Threat Detection Example (cont.)

(All Pairs Testing from ProTest generates 15 test cases)

	Factor_A	Factor_B	Factor_C	Factor_D	Factor_E	Factor_F	Factor_G
Factor Name	Submarine Type	Ocean Depth	Sonar Type	Target Depth	Sea Bottom	Control Mode	Ocean Current
Case 1	S3	Deep	Passive	Very Deep	Mud	Manual	Minimal
Case 2	S1	Very Deep	Passive	Surface	Rock	Autonomous	Strong
Case 3	S2	Shallow	Active	Shallow	Rock	Manual	Moderate
Case 4	S2	Deep	Passive	Deep	Sand	Autonomous	Moderate
Case 5	S1	Shallow	Active	Surface	Sand	Manual	Minimal
Case 6	S1	Very Deep	Passive	Shallow	Mud	Autonomous	Minimal
Case 7	S3	Very Deep	Active	Deep	Mud	Manual	Strong
Case 8	S2	Very Deep	Active	Very Deep	Sand	Autonomous	Strong
Case 9	S3	Shallow	Passive	Shallow	Mud	Autonomous	Strong
Case 10	S3	Deep	Active	Surface	Rock	Manual	Moderate
Case 11	S1	Shallow	Active	Deep	Rock	Autonomous	Minimal
Case 12	S1	Deep	Passive	Very Deep	Rock	Manual	Moderate
Case 13	S2	Very Deep	Active	Surface	Mud	Autonomous	Moderate
Case 14	S3	Deep	Active	Shallow	Sand	Manual	Strong
Case 15	S2	Shallow	Active	Very Deep	Rock	Manual	Minimal

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Command & Control Test Example

(15 factors each at various levels)
Total Combinations: 20,155,392

Variable or Factor	Levels	(# of levels)
Mission Snapshots	Entry, Operations, Consolidation	(3)
Network Size	10 Nodes, 50 Nodes, 100 Nodes	(3)
Network Loading	Nominal, 2X, 4X	(3)
Movement Posture	ATH, OTM1, OTM2	(3)
SATCOM Band	Ku, Ka, Combo	(3)
SATCOM Look Angle	0, 45, 75	(3)
Link Degradation	0%, 5%, 10%, 20%	(4)
Node Degradation	0%, 5%, 10%, 20%	(4)
EW	None, Terrestrial, GPS	(3)
Interoperability	Joint Services, NATO	(2)
IA	None, Spoofing, Hacking, Flooding	(4)
Security	NIPR, SIPIR	(2)
Message Type	Data, Voice, Video	(3)
Message Size	Small, Medium, Large, Mega	(4)
Distance Between Nodes	Short, Average, Long	(3)

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Command & Control Test Example (cont.)

(All Pairs Testing from ProTest generates 26 test cases)

	Factor_A	Factor_B	Factor_C	Factor_D	Factor_E	Factor_F	Factor_G	Factor_H	Factor_I	Factor_J	Factor_K	Factor_L	Factor_M	Factor_N	Factor_O
Factor Name	Mission	Network Size	Network Load	Movement	SATCOM Band	SATCOM Angle	Link Degradation	Node Degradation	EW	Interoperability	IA	Security	Message Type	Size of Message	Node Distance
Case 1	Entry	100 nodes	4X	OTM2	Combo	0	0%	0%	None	NATO	None	SIPIR	Voice	Medium	Short
Case 2	Consolidation	10 nodes	Normal	ATH	Ka	45	5%	5%	GPS	NATO	Spoofing	NIPR	Video	Large	Normal
Case 3	Operation	50 nodes	2X	OTM1	Ku	75	20%	20%	Terrestrial	Joint Serv	Hacking	NIPR	Voice	Small	Long
Case 4	Entry	50 nodes	2X	ATH	Ku	45	10%	10%	None	NATO	Flooding	NIPR	Data	Mega	Short
Case 5	Operation	100 nodes	Normal	OTM1	Combo	75	10%	10%	GPS	NATO	Spoofing	SIPIR	Data	Small	Normal
Case 6	Operation	10 nodes	4X	OTM2	Combo	45	0%	5%	Terrestrial	Joint Serv	None	NIPR	Video	Mega	Long
Case 7	Consolidation	100 nodes	4X	ATH	Ka	75	20%	10%	Terrestrial	NATO	Hacking	SIPIR	Video	Medium	Long
Case 8	Operation	10 nodes	Normal	ATH	Ka	0	20%	0%	Terrestrial	Joint Serv	Flooding	NIPR	Data	Large	Short
Case 9	Consolidation	10 nodes	2X	OTM2	Ku	45	5%	20%	None	Joint Serv	Flooding	SIPIR	Voice	Medium	Normal
Case 10	Consolidation	50 nodes	2X	OTM1	Combo	0	0%	20%	GPS	NATO	None	NIPR	Data	Mega	Normal
Case 11	Entry	50 nodes	Normal	OTM2	Ka	75	10%	5%	GPS	Joint Serv	Hacking	SIPIR	Voice	Large	Long
Case 12	Entry	50 nodes	4X	OTM1	Ku	0	5%	0%	None	Joint Serv	Spoofing	SIPIR	Video	Small	Long
Case 13	Consolidation	100 nodes	4X	OTM2	Ku	45	20%	5%	GPS	Joint Serv	Flooding	NIPR	Data	Small	Short
Case 14	Entry	10 nodes	2X	OTM1	Ka	75	5%	0%	None	Joint Serv	Hacking	SIPIR	Data	Mega	Normal
Case 15	Entry	50 nodes	2X	ATH	Ka	75	0%	20%	Terrestrial	NATO	Spoofing	NIPR	Video	Large	Short
Case 16	Consolidation	10 nodes	4X	ATH	Ku	0	10%	20%	Terrestrial	NATO	None	NIPR	Video	Small	Normal
Case 17	Operation	50 nodes	Normal	OTM1	Ku	75	0%	5%	None	Joint Serv	Flooding	NIPR	Data	Medium	Short
Case 18	Operation	10 nodes	Normal	OTM1	Ka	75	20%	10%	None	Joint Serv	None	SIPIR	Video	Large	Normal
Case 19	Operation	100 nodes	2X	OTM2	Combo	0	5%	10%	Terrestrial	NATO	Hacking	SIPIR	Data	Large	Short
Case 20	Consolidation	100 nodes	Normal	ATH	Combo	0	20%	20%	Terrestrial	Joint Serv	Spoofing	NIPR	Voice	Mega	Short
Case 21	Consolidation	50 nodes	2X	OTM1	Ka	45	10%	0%	GPS	Joint Serv	Spoofing	SIPIR	Data	Medium	Normal
Case 22	Entry	100 nodes	Normal	OTM1	Combo	0	20%	5%	GPS	NATO	Flooding	NIPR	Video	Medium	Long
Case 23	Operation	10 nodes	Normal	ATH	Ka	45	0%	10%	None	NATO	Hacking	SIPIR	Voice	Small	Normal
Case 24	Entry	50 nodes	4X	ATH	Ku	45	5%	20%	None	NATO	None	NIPR	Video	Large	Long
Case 25	Consolidation	10 nodes	2X	ATH	Ku	75	10%	5%	None	Joint Serv	Spoofing	NIPR	Data	Large	Long
Case 26	Consolidation	100 nodes	Normal	OTM2	Combo	45	5%	20%	GPS	Joint Serv	Spoofing	NIPR	Voice	Mega	Normal

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The Efficiency of All Pairs Testing

- Suppose we had 75 Factors to test.

- Suppose we wanted to test each of these at 2 levels.

- How many total combinations are there?

  2^{75} = 37, 778, 931, 862, 957, 161, 709, 568

  i.e., 37 Sextillion, 778 Quintillion, 931 Quadrillion, 862 Trillion, 957 Billion, 161 Million, 709 Thousand, 568

- What is the minimum number of these combinations that will have to be tested in order to test every 2-way combination?

- To answer this question, we used Pro-Test software. The answer is 14 runs or experimental test combinations.

- For k factors each having the same number of levels tested, say v, then the minimum number of tests $\approx v^2$ (ln k)

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HTT Applications

- Reducing the cost and time of testing while maintaining adequate test coverage
- Integration, functionality, or validation testing
- Creating a test plan to stress a product and discover problems
- Prescreening before a large DOE to ensure all 2-way combinations are feasible before discovering, midway through an experiment, that certain combinations are not feasible
- Developing an “outer array” of noise combinations to use in a robust design DOE when the number of noise factors and settings is large

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Key Take-Aways

- DOE brings orthogonal or nearly orthogonal designs into play.
- Various approaches to combinatorial test, to include OFAT and Oracle.
- Orthogonality is key to being able to evaluate the effects of factors and their interactions independently from one another. It also connects test and analysis (Scientific Test and Analysis Techniques – STAT).
- Factorial designs are great, but in a world of large test design spaces, we need something else.
- Nearly Orthogonal Latin Hypercube Designs provide a sampling strategy to test a large number of factors with a much smaller number of runs than what a factorial design requires, while still retaining adequate orthogonality. In NOLHDs, each factor is tested at the same number of levels (typically at least 5 levels).
- All Pairs Testing, a special instance of High Throughput Testing, is a way to get great test coverage (i.e., all 2-way combinations) with a minimal number of runs when the test scenario involves mixed factors and mixed levels.

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Thank You

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