Full Factorial Designs (2 Levels)

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Full Factorial Designs (2 Levels)

- In this session, we will discuss:
  - What is a Full Factorial Design?
  - Advantages and Disadvantages of Full Factorials
  - Estimating Effects and What is an Interaction?
  - Orthogonality and Coding of Design Matrices
  - Analysis of a Full Factorial Design
    - Graphical Analysis
      - marginal means plot, interaction plot, pareto, surface/contour plot
    - Statistical Analysis
      - regression, p-values, R-squared value, Adjusted R-squared
    - Prediction Equations
    - Optimization
  - Examples of Full Factorial designs
  - Practice analyzing the data with DOE Pro software and drawing conclusions

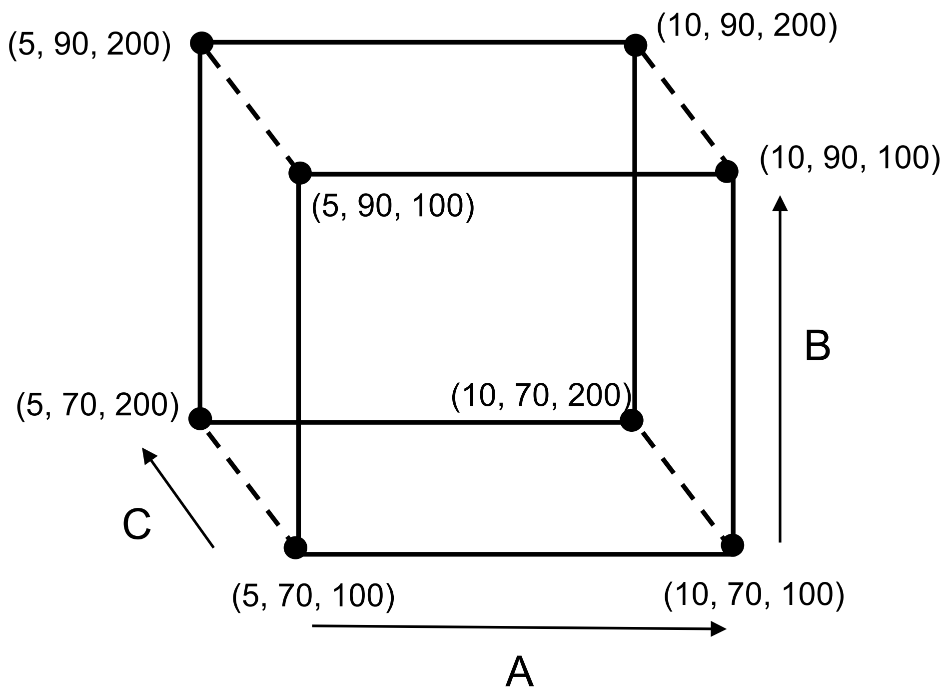
Take Note

- A list of supplemental material and additional practice/review questions for this session are provided at the end of this presentation
- You can download the pdf of this presentation, along with any supporting data files, on the site where you are accessing this course

What is a Full Factorial Design?

- A full factorial is a design where we test all the possible combinations.
- Example:
  - 3 factors (A: time, B: temp, C: pressure)
  - 2 settings for each: (A: 5, 10) (B: 70, 90) (C: 100, 200)
  - The full factorial consists of 8 test combinations (runs)
  - # runs = (# of settings)$^{\text{\# of factors}}$ = 2^3 = 8

Run	Time	Temp	Pressure
1	5	70	100
2	5	70	200
3	5	90	100
4	5	90	200
5	10	70	100
6	10	70	200
7	10	90	100
8	10	90	200



- Testing at 2 settings (levels) will produce a linear model. As with all DOEs, confirmation is critical to validate assumptions.

Full Factorial Designs

- Advantages
  - Orthogonal (balanced vertically and horizontally)
  - Test and learn about all the possible combinations, including <u>interaction</u> effects
  - Lots of knowledge gained (effects of factors, prediction, optimization)
- Disadvantages
  - Requires a lot of tests if there are a large number of factors (so typically we will conduct screening tests first to help reduce the number of factors)

Orthogonal:
- Vertical balance within a column
- Horizontal balance between columns

Orthogonality allows us to estimate effects independently!

Run	A	B	C
1	5	70	100
2	5	70	200
3	5	90	100
4	5	90	200
5	10	70	100
6	10	70	200
7	10	90	100
8	10	90	200

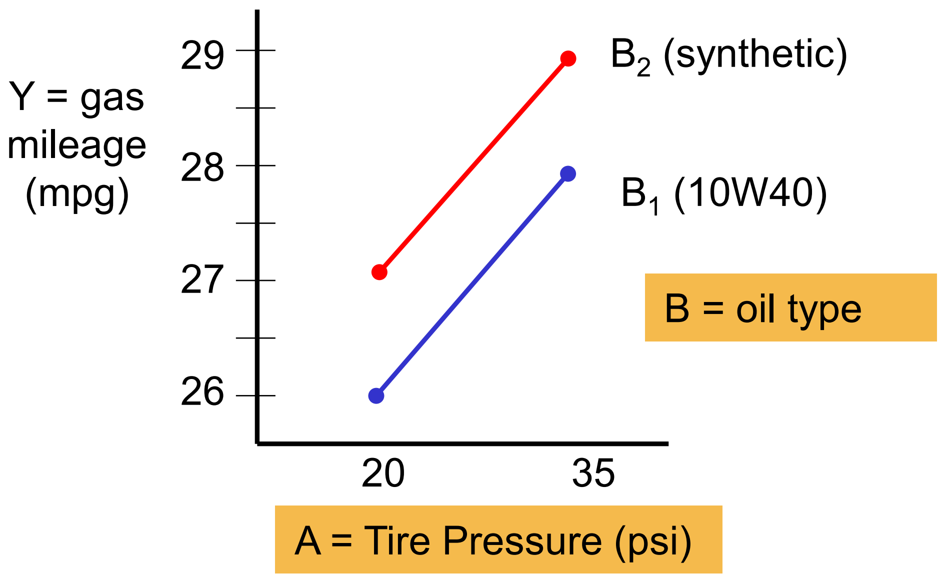
The columns will be used to determine the “effect” of each factor.

We’ll compare the average response at the low setting of a factor with the average response at the high setting of a factor (upcoming example)

What is an Interaction?

- An interaction between two factors means that the effect that one factor has on the response depends on the setting of another factor

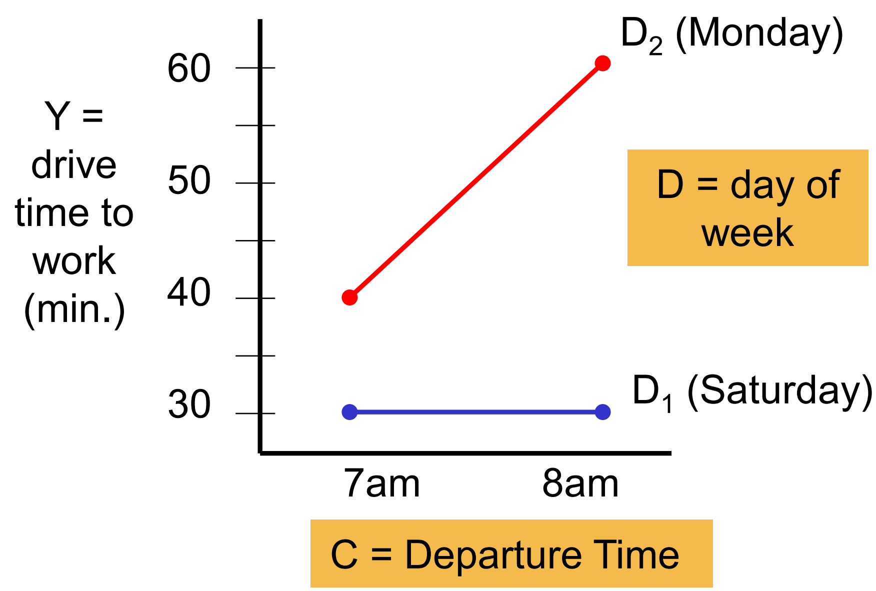
- Illustration with NO interaction



Q: What effect does tire pressure (A) have on gas mileage? "increase of 2 mpg"

Q: What effect does oil type (B) have on gas mileage? "increase of 1 mpg"

- Illustration WITH interaction



Q: What effect does departure time (C) have on drive time? "it depends"

Q: What effect does day of week (D) have on drive time? "it depends"

Estimating Interaction Effects

- For running the experiment, the design matrix gives us the “recipe” of test settings
- When analyzing the data, however, additional columns are created behind-the-scenes, which allow us to estimate the interaction effects

Design Matrix
(For running the experiment)

Run	A	B	C
1	5	70	100
2	5	70	200
3	5	90	100
4	5	90	200
5	10	70	100
6	10	70	200
7	10	90	100
8	10	90	200

AB
350
350
450
450
700
700
900
900

- For example, what if we want to learn about the AB interaction?
- Mathematically, the column to estimate the interaction effect is created by multiplying the A and B columns as shown.
- Is this AB column orthogonal with column A?

Orthogonality and Coding

- For <u>analysis</u> purposes, coding of the design matrix is done by software
- -1 is used for the low setting of a factor, while +1 is used for the high setting
- Coding allows us to:
  - Evaluate all effects independently, including interactions
  - Put everything on the same scale (for comparison of effects)

For running the experiment

Run	**A**	**B**	**C**
1	5	70	100
2	5	70	200
3	5	90	100
4	5	90	200
5	10	70	100
6	10	70	200
7	10	90	100
8	10	90	200

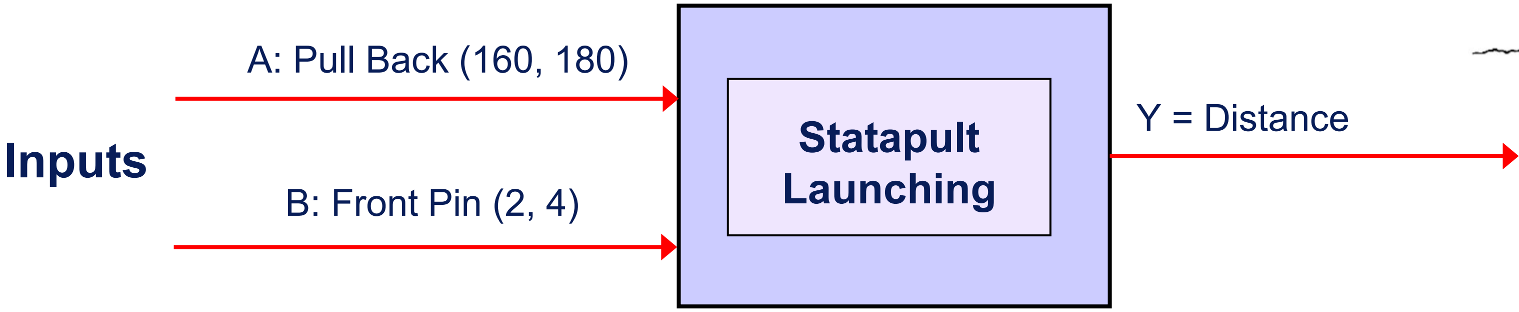
For analyzing the experiment

Coded Design Matrix

A	B	C	AB	AC	BC	ABC
-1	-1	-1	1	1	1	-1
-1	-1	1	1	-1	-1	1
-1	1	-1	-1	1	-1	1
-1	1	1	-1	-1	1	-1
1	-1	-1	-1	-1	1	1
1	-1	1	-1	1	-1	-1
1	1	-1	1	-1	-1	-1
1	1	1	1	1	1	1

Example: 2 Factor Full Factorial

- To see how the analysis works, let's start with a simple 2-factor example
- In a statapult launching process, the pull back angle and front pin position were both varied, and there were 3 replicates. The results of the experiment are shown below:



Run	Pull Back	Front Pin		Launch Distance (Y)				Avg. ($\bar{y}$)	Std Dev (s)
				y_1	y_2	y_3			
1	160	2		63	61	62		62	1
2	160	4		97	99	98		98	1
3	180	2		91	94	97		94	3
4	180	4		140	148	144		144	4

- The goal is to discover how pull back angle and front pin affect distance, and learn how to meet the customer requirement (target distance = <u>115 inches +/- 6 inches</u>)

for background / reference . . . don't worry, software will do this for us!

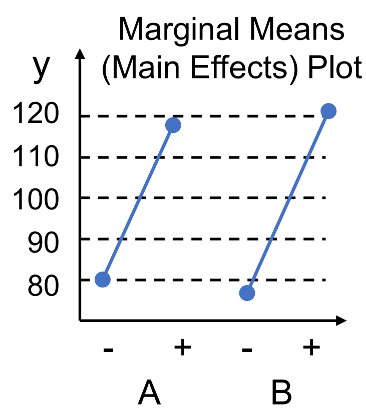
Example: 2 Factor Full Factorial (Analysis) (y-hat)

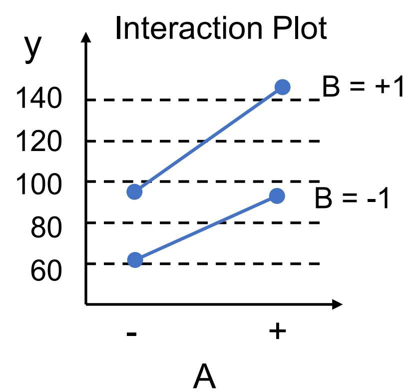
Run	Pull Back	Front Pin	A	B	AB		Avg. ($\bar{y}$)	Std Dev (s)
1	160	2	-1	-1	1		62	1
2	160	4	-1	1	-1		98	1
3	180	2	1	-1	-1		94	3
4	180	4	1	1	1		144	4
Analysis for Average ($\bar{y}$)		Avg @ -	80	78	96			
		Avg @ +	119	121	103			
(Avg @ +) - (Avg @ -)		Δ	+39	+43	+7			

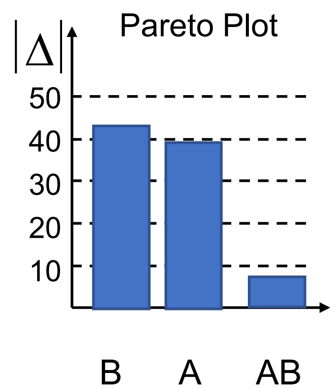
$\bar{\bar{y}}$ = overall average = $\frac{62 + 98 + 94 + 144}{4}$ = 99.5

$\hat{y} = \bar{\bar{y}} + \frac{\Delta_A}{2}(A) + \frac{\Delta_B}{2}(B) + \frac{\Delta_{AB}}{2}(AB)$

$= 99.5 + \frac{+39}{2}A + \frac{+43}{2}B + \frac{+7}{2}AB = 99.5 + 19.5A + 21.5B + 3.5AB$







Prediction Equation

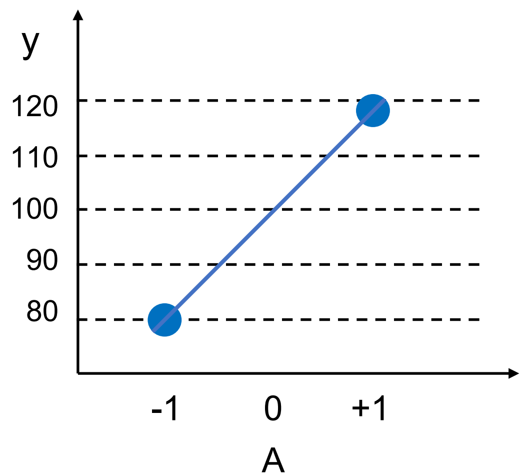
$$\hat{y} = \bar{\bar{y}} + \frac{\Delta_A}{2}A + \frac{\Delta_B}{2}B + \frac{\Delta_{AB}}{2}A \bullet B + \ldots$$

Where does the Prediction Equation come from?

Assumptions

1. Orthogonal design
2. Orthogonal coding (-1, +1)
3. 2 levels only

Prediction Equation for 1 Factor (A)



$\hat{y} = 19.5A + 99.5$

<u>Slope Intercept Equation of a Line</u>

$y = mx + b$

$$m = \text{slope} = \frac{\text{rise}}{\text{run}} = \frac{119 - 80}{(1 - (-1))} = \frac{39}{2} = \frac{\Delta}{2}$$

$b = \text{y-intercept} = \bar{\bar{y}}$

Justification for adding on other factors: orthogonality for background / reference . . . don't worry, software will do this for us!

Example: 2 Factor Full Factorial (Analysis) (with s-hat)

Run	Pull Back	Front Pin	A	B	AB		Avg. ($\bar{y}$)	Std Dev (s)
1	160	2	-1	-1	1		62	1
2	160	4	-1	1	-1		98	1
3	180	2	1	-1	-1		94	3
4	180	4	1	1	1		144	4

$\bar{\bar{y}}$ = overall average = $\frac{62 + 98 + 94 + 144}{4}$ = 99.5

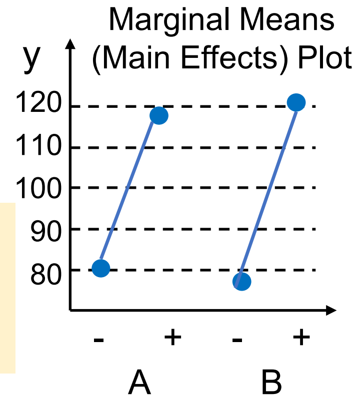
$\bar{s}$ = overall average = $\frac{1 + 1 + 3 + 4}{4}$ = 2.25

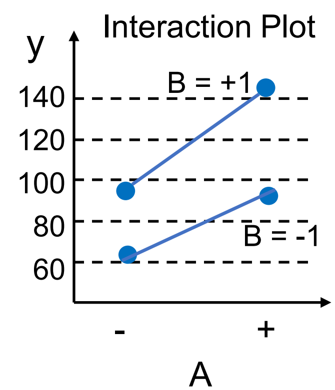
Analysis for Average ($\bar{y}$)

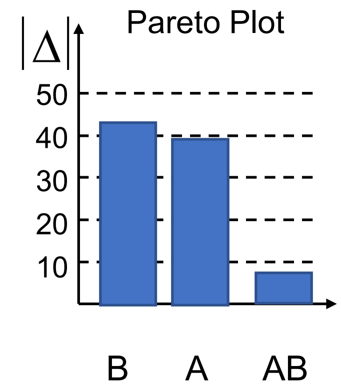
	A	B	AB
Avg @ -	80	78	96
Avg @ +	119	121	103
Δ	+39	+43	+7

$\hat{y} = \bar{\bar{y}} + \frac{\Delta_A}{2}(A) + \frac{\Delta_B}{2}(B) + \frac{\Delta_{AB}}{2}(AB)$

$= 99.5 + \frac{+39}{2}A + \frac{+43}{2}B + \frac{+7}{2}AB = 99.5 + 19.5A + 21.5B + 3.5AB$





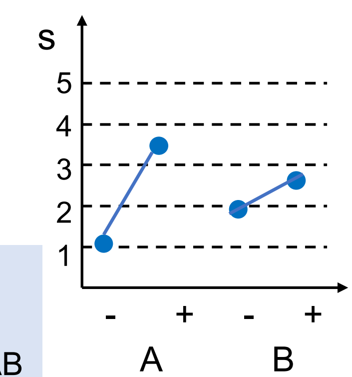


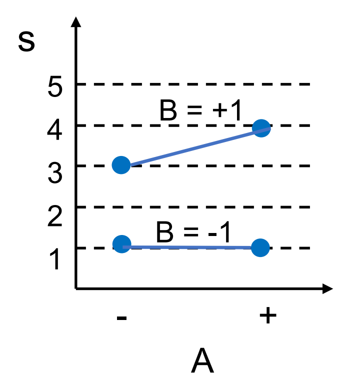
Analysis for Std Dev (s)

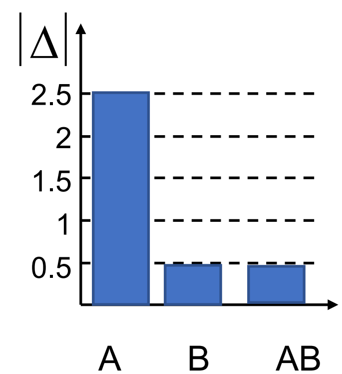
	A	B	AB
Avg @ -	1	2	2
Avg @ +	3.5	2.5	2.5
Δ	+2.5	+0.5	+0.5

$\hat{s} = \bar{s} + \frac{\Delta_A}{2}(A) + \frac{\Delta_B}{2}(B) + \frac{\Delta_{AB}}{2}(AB)$

$= 2.25 + \frac{+2.5}{2}A + \frac{+0.5}{2}B + \frac{+0.5}{2}AB = 2.25 + 1.25A + 0.25B + 0.25AB$





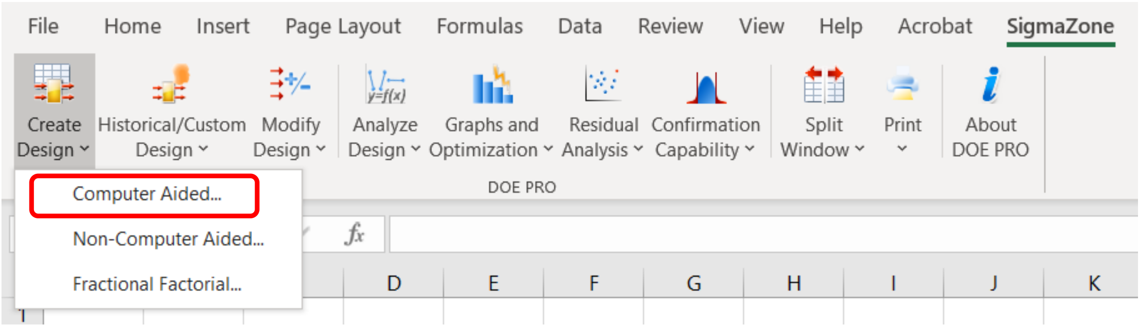


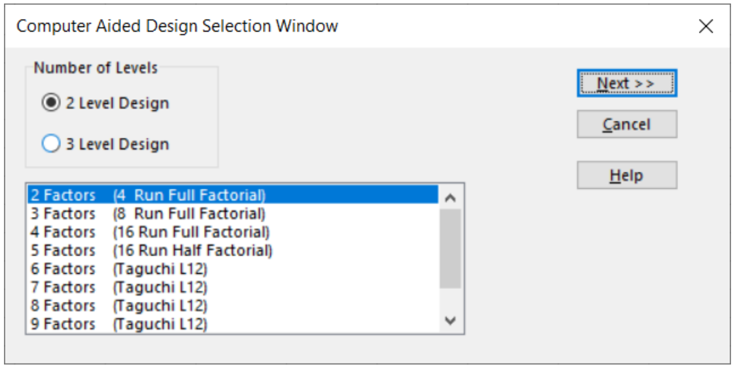
Analysis Using DOE Pro Software

- Let's set up and analyze the 2-factor statapult DOE using DOE Pro software

- First, set up the design matrix. From the SigmaZone (DOE Pro) ribbon:

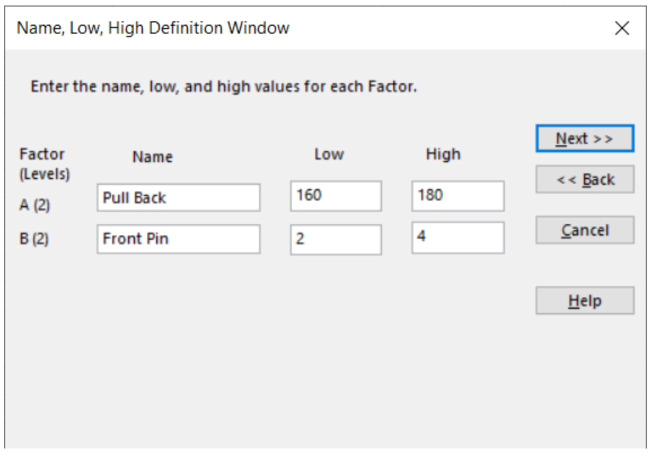
  Create Design > Computer Aided . . .

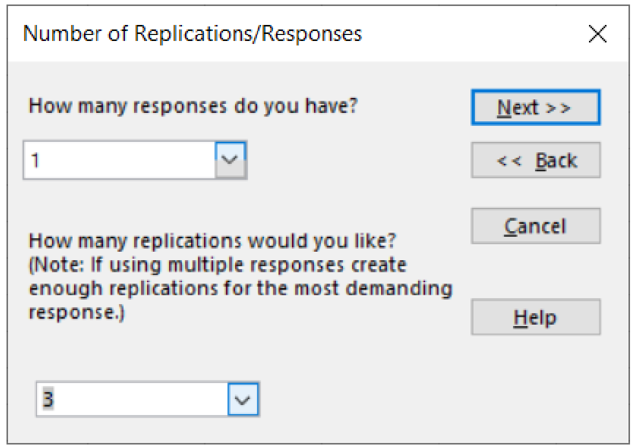


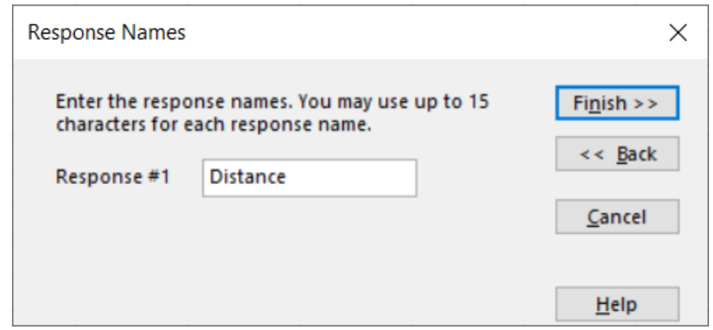


Analysis Using DOE Pro Software (cont.)

- Name the factors and specify the low and high settings
- Specify the number of responses (1 for this example, distance) and the number of replications (3 for this example)
- Name the response (distance for this example)







- When finished setting up the design, input the data into the template as shown:

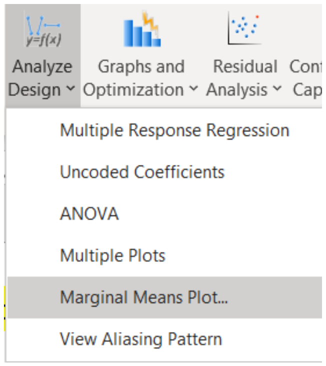
A	B	C	D	E	F	G	H	I	J
Factor	A	B		Distance					
Row #	Pull Back	Front Pin		Y1	Y2	Y3		Y bar	S
1	160	2		63	61	62		62	1
2	160	4		97	99	98		98	1
3	180	2		91	94	97		94	3
4	180	4		140	148	144		144	4

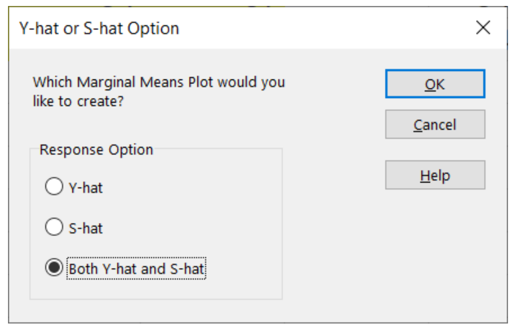
Data file: DOE 2 factor statapult.xlsx

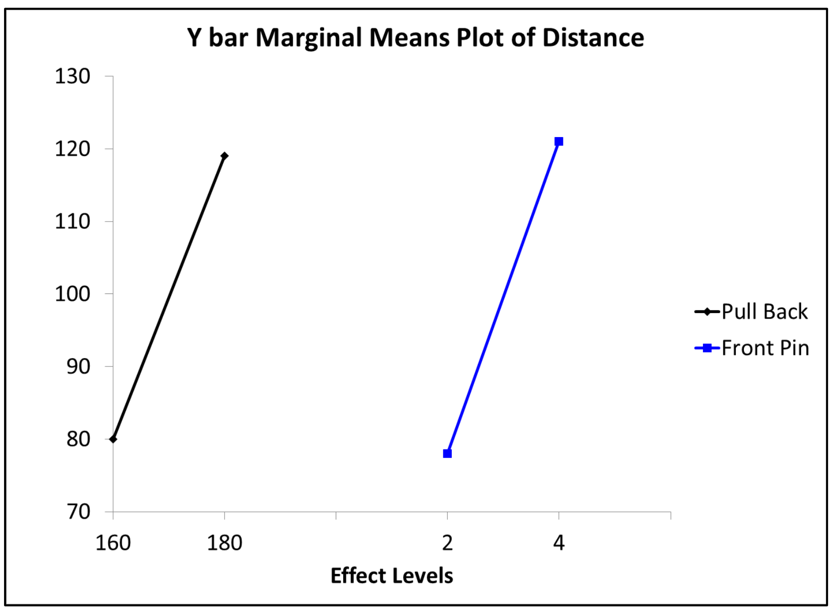
Analysis Using DOE Pro Software (Marginal Means Plots)

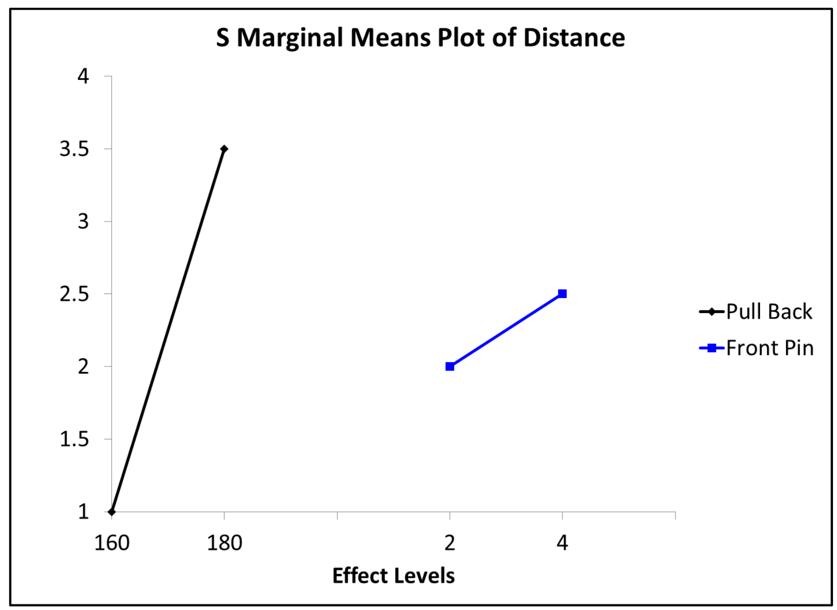
- To create the marginal means plots, from the SigmaZone (DOE Pro) ribbon:

<u>Analyze Design > Marginal Means Plot. . .</u>





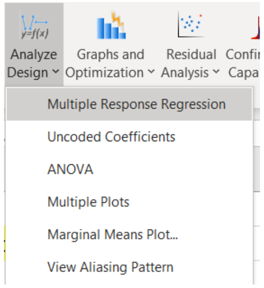




Analysis Using DOE Pro Software (Regression Output)

- To create the regression output (y-hat and s-hat models), from the SigmaZone (DOE Pro) ribbon:

<u>Analyze Design > Multiple Response Regression</u>



Y-hat Model		Distance			
Factor	Name	Coeff	P(2 Tail)	Tol	Active
Const		99.500	0.0000		
A	Pull Back	19.500	0.0000	1	X
B	Front Pin	21.500	0.0000	1	X
AB		3.500	0.0016	1	X
	R^2	0.9948			
	Adj R^2	0.9928			
	Std Error	2.5981			
	F	506.5185			
	Sig F	0.0000			
	F_{LOF}	NA			
	Sig F_{LOF}	NA			
	Source	SS	df	MS	
	Regression	10257.0	3	3419.0	
	Error	54.0	8	6.8	
	$Error_{Pure}$	54.0	8	6.8	
	$Error_{LOF}$	0.0	0	NA	
	Total	10311.0	11		

Factor	Name	Low	High	Exper
A	Pull Back	160	180	170
B	Front Pin	2	4	3

Multiple Response Prediction				
			99% Confidence Interval	
	Y-hat	S-hat	Lower Bound	Upper Bound
Distance	99.5000	2.2500	92.750	106.250

S-hat Model		Distance			
Factor	Name	Coeff	P(2 Tail)	Tol	Active
Const		2.250	NA		
A	Pull Back	1.250	NA	1	X
B	Front Pin	0.25000	NA	1	X
AB		0.25000	NA	1	X
	R^2	1.0000			
	Adj R^2	NA			
	Std Error	NA			
	F	NA			
	Sig F	NA			
	F_{LOF}	NA			
	Sig F_{LOF}	NA			
	Source	SS	df	MS	
	Regression	6.8	3	2.3	
	Error	0.0	0	NA	
	$Error_{Pure}$	NA	0	NA	
	$Error_{LOF}$	NA	0	NA	
	Total	6.8	3		

Determining Statistical Significance

- **p-values** (column labeled P(2 tail) in DOE Pro)
  - tells which effects are significant
  - ROT:
    - If **p(2 tail) < 0.05**, this is a highly significant effect (leave in model)
    - If **0.05 < p(2 tail) < 0.10**, this is a moderately significant effect (use judgment, although usually leave in model)
    - If **p(2 tail) > 0.10**, this indicates an insignificant effect (remove from the model)

- **Rule of Hierarchy** when building models
  - If an interaction (example: AB) or higher order term (example: A^2) is significant, then include all main effects involved in that term in the model regardless of their significance
  - Examples:
    - If AB is significant, then include A and B in the model
    - If A^2 is significant, then include A in the model

- **s-hat models with no p-values** (for 2-level designs ONLY)
  - ROT: Any coefficient (in absolute value) greater than or equal to half the constant term is considered statistically significant (leave in the model)
  - Note: this is an approximate rule of thumb, designed to provide at least 95% confidence. It should not be treated as a razor sharp cutoff. If a coefficient is large and “close” in size to half the constant, try leaving it in the model and re-running the regression to obtain a p-value.

Assessing the Regression Model Fit

- R-squared Value
  - Measure of "goodness of fit"
  - Scale: from 0 to 1
  - Measures the proportion of variation that is being explained by the regression model

- Adjusted R-squared Value
  - Modified measure of R-squared
  - Adjusts for sample size and/or too many terms in the model
  - More realistic measure of the explanatory value of the model
  - ROT: Want to see the adjusted R-squared value close to the R-squared value (ideally, not less than 90% of R-squared)

- Standard Error
  - Measures the variation (standard deviation) about the regression line
  - This is used as an estimate for standard deviation (s) when there is no s-hat model available

- F value
  - This provides an overall test of the significance of the model.
  - ROT: values over 6 indicate a significant model (i.e., not all regression coefficients are equal to zero). SigF provides a p-value for the test, with values less than 0.05 indicating that the overall model is significant.

Analysis Using DOE Pro Software (Reduced Regression)

- After re-running the regression to remove the insignificant terms, the resulting output is shown below
- In this example, only B and AB in the s-hat model were insignificant and thus removed from the model

Y-hat Model — Distance

Factor	Name	Coeff	P(2 Tail)	Tol	Active
Const		99.500	0.0000		
A	Pull Back	19.500	0.0000	1	X
B	Front Pin	21.500	0.0000	1	X
AB		3.500	0.0016	1	X

R^2	0.9948
Adj R^2	0.9928
Std Error	2.5981
F	506.5185
Sig F	0.0000
F_{LOF}	NA
Sig F_{LOF}	NA

Source	SS	df	MS
Regression	10257.0	3	3419.0
Error	54.0	8	6.8
$Error_{Pure}$	54.0	8	6.8
$Error_{LOF}$	0.0	0	NA
Total	10311.0	11	

Factor	Name	Low	High	Exper
A	Pull Back	160	180	170
B	Front Pin	2	4	3

Multiple Response Prediction

	Y-hat	S-hat	99% Confidence Interval Lower Bound	99% Confidence Interval Upper Bound
Distance	99.5000	2.2500	92.750	106.250

S-hat Model — Distance

Factor	Name	Coeff	P(2 Tail)	Tol	Active
Const		2.250	0.0121		
A	Pull Back	1.250	0.0377	1	X

R^2	0.9259
Adj R^2	0.8889
Std Error	0.5000
F	25.0000
Sig F	0.0377
F_{LOF}	NA
Sig F_{LOF}	NA

Source	SS	df	MS
Regression	6.3	1	6.3
Error	0.5	2	0.3
$Error_{Pure}$	0.5	2	0.3
$Error_{LOF}$	0.0	0	NA
Total	6.8	3	

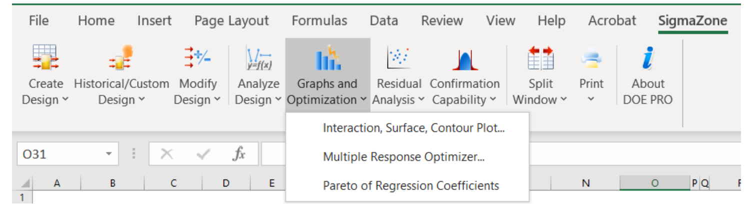
- The prediction models, in coded units, are:

$$\hat{y} = 99.5 + 19.5A + 21.5B + 3.5AB$$

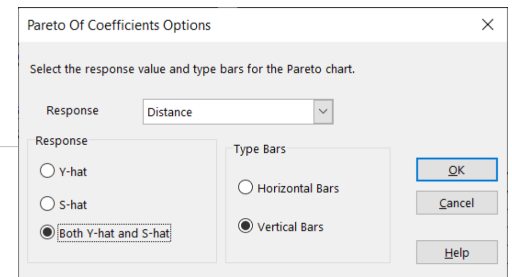
$$\hat{s} = 2.25 + 1.25A$$

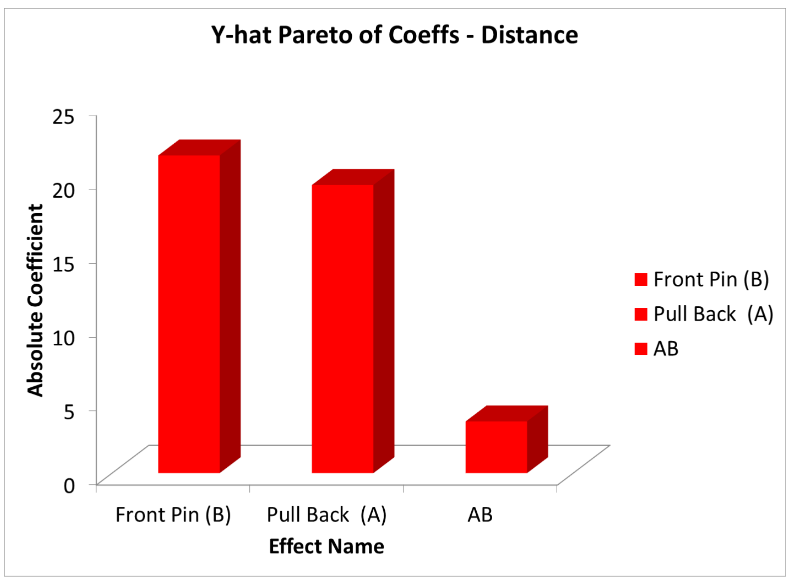
Analysis Using DOE Pro Software (Other Plots)

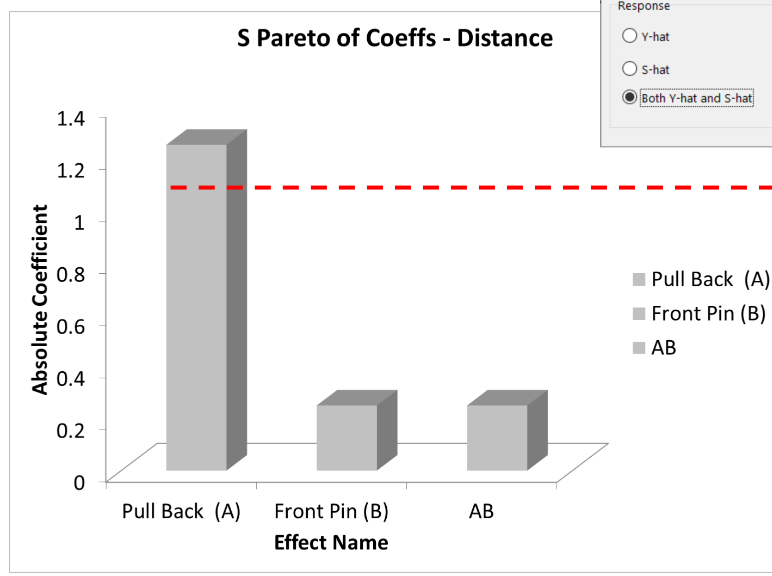
- Other graphical outputs available include:
  - Interaction, Surface, Contour Plots
  - Pareto of Regression Coefficients



- The pareto of the regression coefficients is a graphical summary which compares the magnitude (in absolute value) of the regression coefficients. The color coding is associated with the p-value (e.g., bars in red are associated with terms having a p-value value below 0.05)





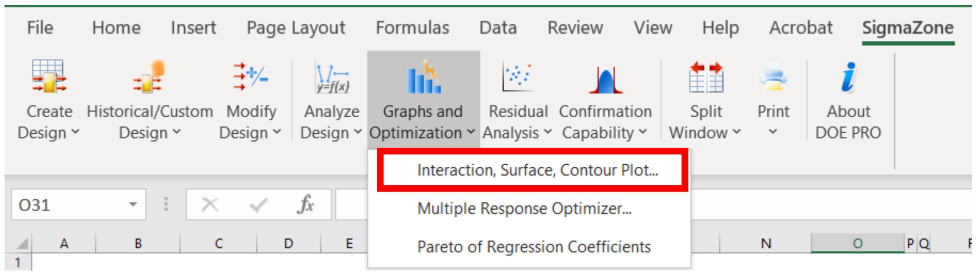


Half the constant:
2.25 / 2 = 1.125

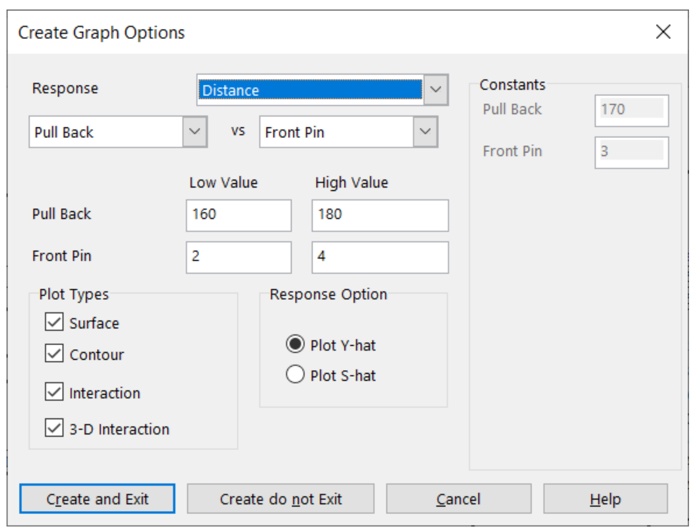
Analysis Using DOE Pro Software (Interaction Plots)

- To view an interaction plot and other graphical output, from the SigmaZone (DOE Pro) ribbon select:

<u>Graphs and Optimization > Interaction, Surface, Contour Plot . . .</u>

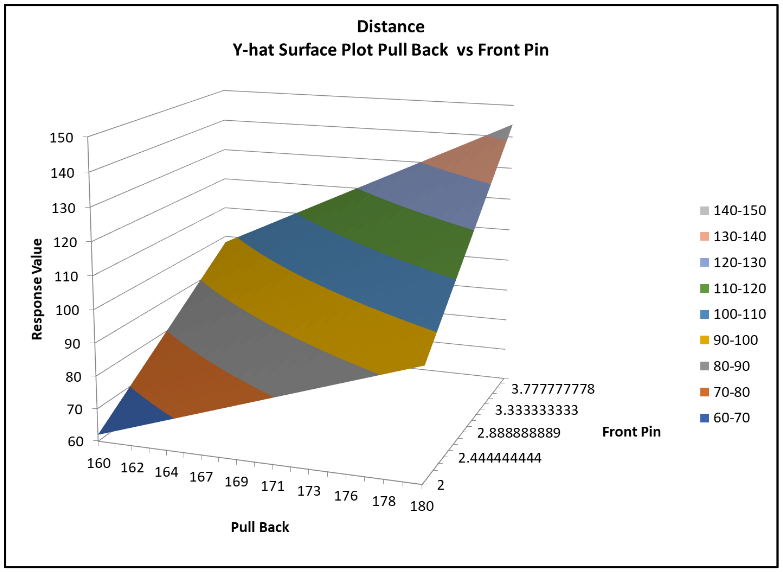


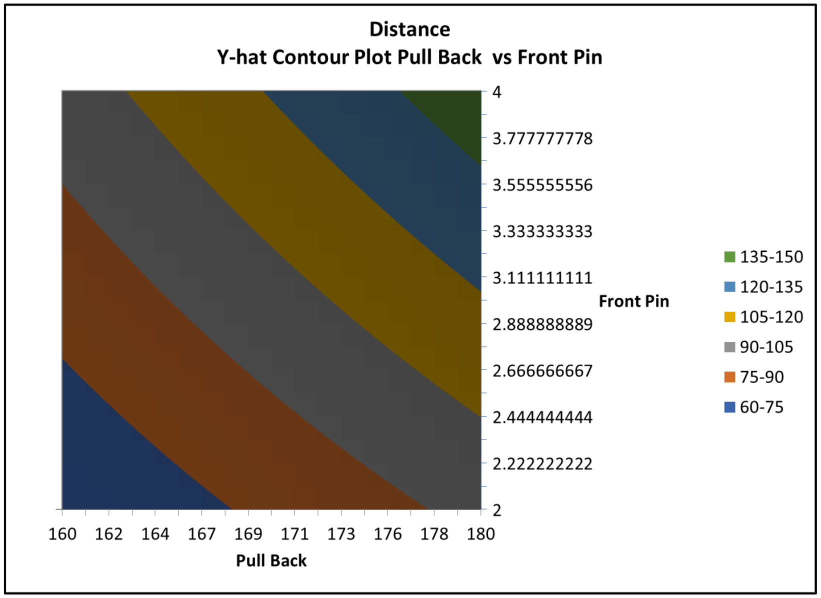
- Select the response of interest (distance in our example) and two of the input factors to plot (pull back angle and front pin in our example). Note: if there are other factors, they will be held constant at the settings specified at the right-hand side of the input dialog box

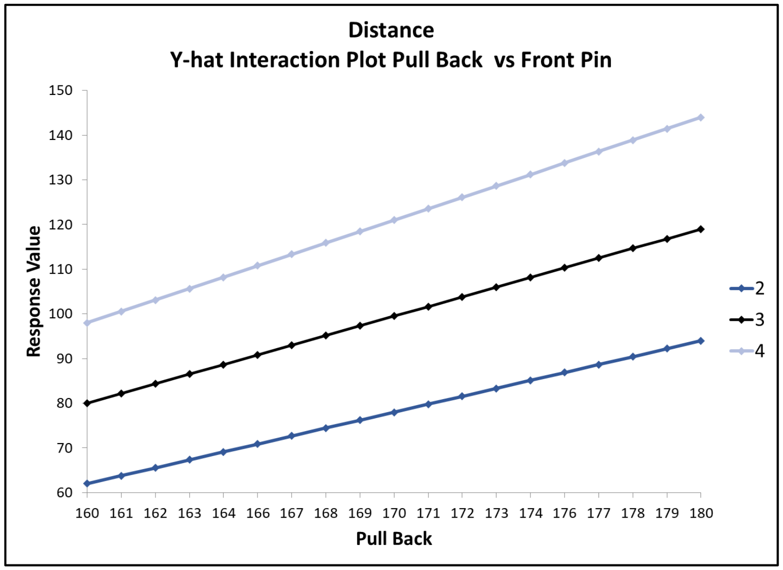


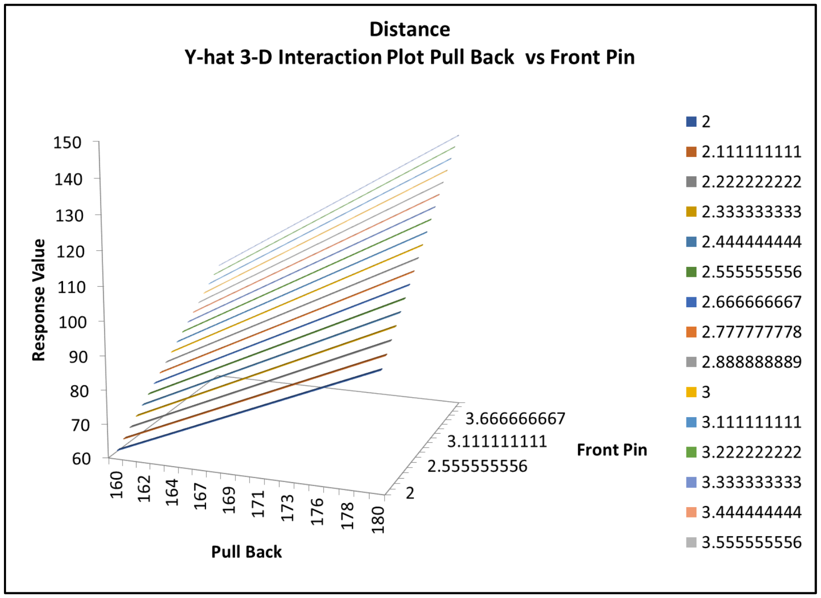
Analysis Using DOE Pro Software (Surface, Contour, Interaction Plots)

- An example of each plot is shown









Analysis Using DOE Pro Software (Making Predictions)

- The center portion of the regression output is used for making predictions
- Type settings into the yellow highlighted area. For example, with a pull back angle equal to 170 degrees and the front pin setting at 3, the predicted distance is 99.5 inches with a standard deviation of 2.25 inches.

Factor	Name	Low	High	Exper
A	Pull Back	160	180	170
B	Front Pin	2	4	3

Multiple Response Prediction				
			99% Confidence Interval	
	Y-hat	S-hat	Lower Bound	Upper Bound
Distance	99.5000	2.2500	92.750	106.250

The 99% confidence interval is for individual launches.

This says that 99% of the launches at this setting should fall between 92.75 and 106.25 inches. This is a prediction for the "center point", and should always be tested to ensure the assumption of a linear model is valid!!

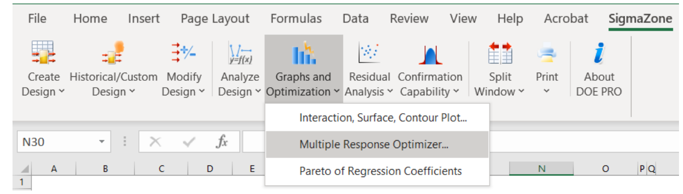
- What is the predicted launch distance if we used a pull back angle of 175 inches and a front pin setting of 2?

Factor	Name	Low	High	Exper
A	Pull Back	160	180	175
B	Front Pin	2	4	2

Multiple Response Prediction				
			99% Confidence Interval	
	Y-hat	S-hat	Lower Bound	Upper Bound
Distance	86.0000	2.8750	77.375	94.625

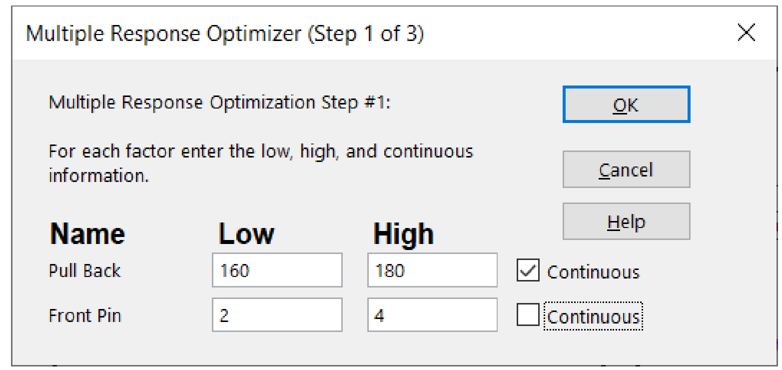
Analysis Using DOE Pro Software (Optimization)

- To determine optimum settings for the factors in order to meet goals, from the SigmaZone (DOE Pro) ribbon, select:

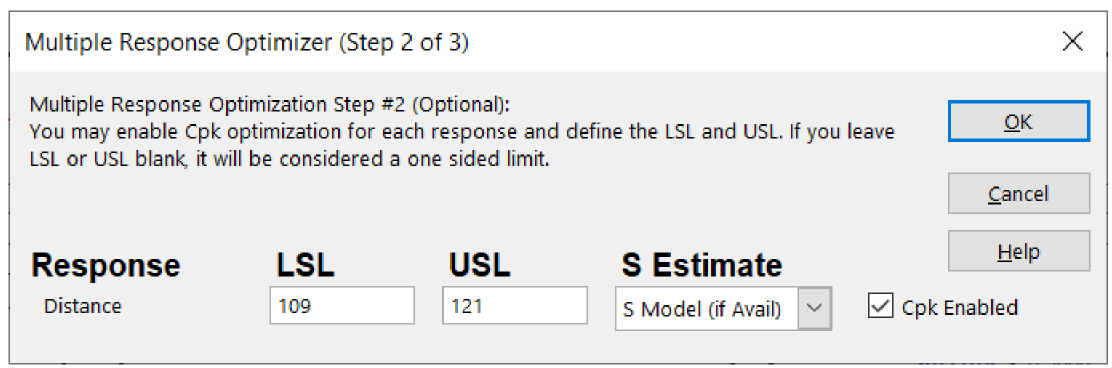


<u>Graphs and Optimization > Multiple Response Optimizer . . .</u>

- **Step 1:** Specify the factor ranges. If a factor can assume any value on a continuous scale, leave a checkmark in the "Continuous" box. If only integer settings are allowed, such as for the front pin setting, be sure to uncheck the "Continuous" box for those factors.

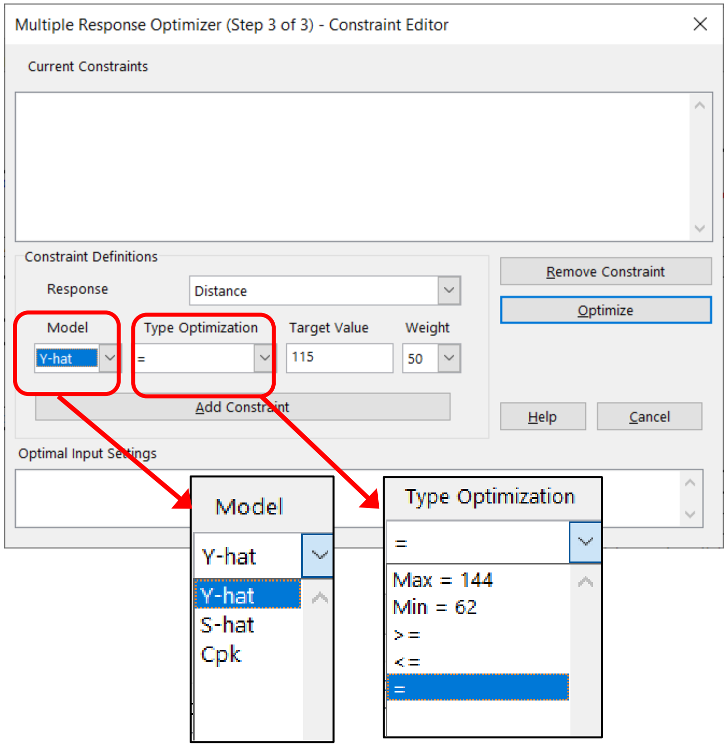


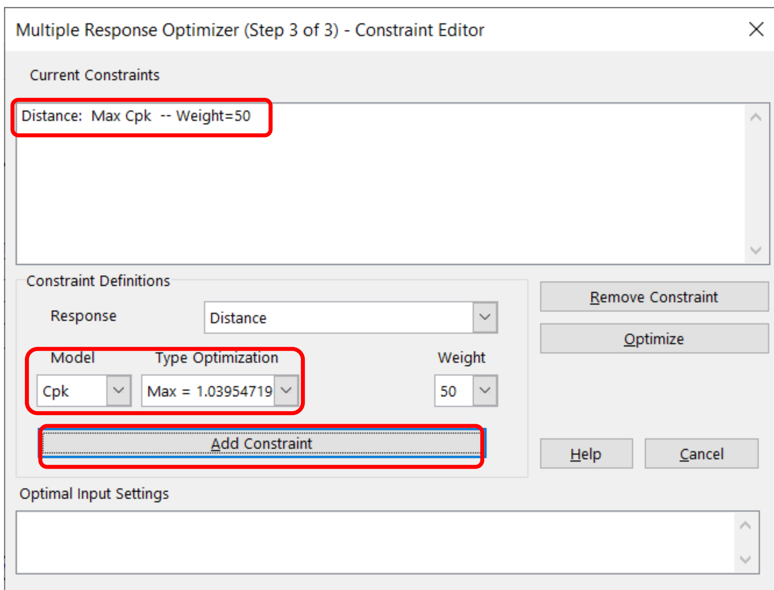
- **Step 2:** If you have one or more spec limits, then check the box "Cpk Enabled" and input the spec limit(s). This will allow for Cpk optimization in the next step.



Analysis Using DOE Pro Software (Optimization) (cont.)

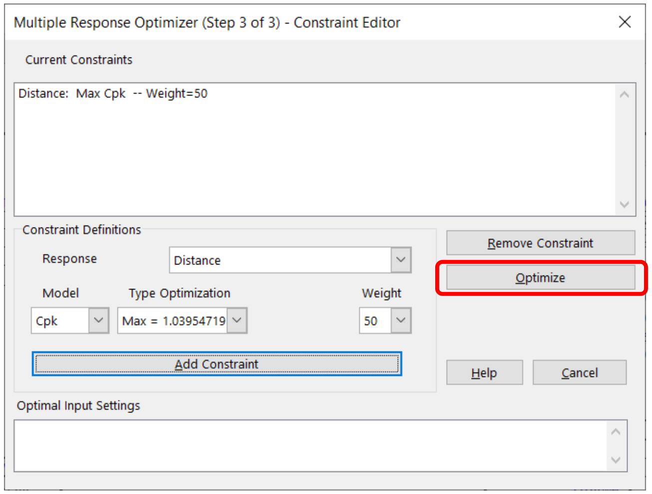
- **Step 3: Specify the constraints for optimization.**
  - For the <u>Model</u>, you can choose y-hat, s-hat, and Cpk (if there are spec limits)
  - For the <u>type of optimization</u>, you can choose to maximize, minimize, make >=, make <=, or make equal to a particular value.
  - Click “Add Constraint” to add a constraint for the optimization. When finished, click “Optimize” to see the optimal values. Click on “Optimize Again” to see if there are other optimal settings.
- **Notes:**
  - You can have multiple constraints
  - Don’t ignore information in any s-hat model (example: Suppose we just ask to hit a target distance of 115. Try it! There are 2 options, but is one better for reducing variation?)
  - Constraints can compete against each other. Weights can be added, optionally, for importance.
  - Simpler is better in terms of the number of constraints. For example, rather than asking to hit a target distance and minimize standard deviation, ask to maximize Cpk (or make the Cpk at least a certain value). Cpk considers both the mean and the standard deviation!



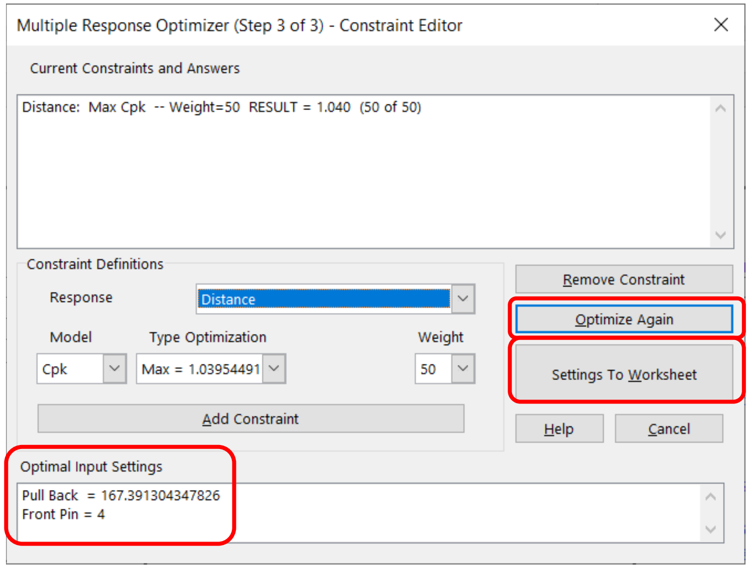


Analysis Using DOE Pro Software (Optimization) (cont.)

- **Step 4: Determine Optimum settings.**
  - Click “Optimize” to see the optimal input settings
  - Sometimes there can be more than one combination to achieve the desired constraints, so click “Optimize Again” to check if there are other settings (sometimes you may have to do this more than once)



- **Step 5: Copy the optimal settings to the worksheet**
  - Once you have your optimal settings, click “Settings to Worksheet” to copy these settings into the prediction area of the regression output, to see the predicted values.



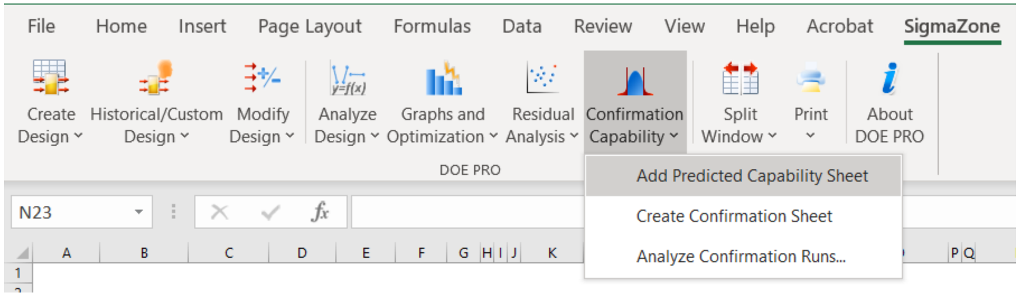
Factor	Name	Low	High	Exper
A	Pull Back	160	180	167.3913043
B	Front Pin	2	4	4

Multiple Response Prediction				
			99% Confidence Interval	
	Y-hat	S-hat	Lower Bound	Upper Bound
Distance	115.0000	1.9239	109.228	120.772

Analysis Using DOE Pro Software (Predicted Capability)

- To add a predicted capability sheet for the proposed settings, from the SigmaZone U(DOE Pro) ribbon, select:

Confirmation Capability > Add Predicted Capability Sheet



Using these settings . . .

Factor	Name	Low	High	Exper
A	Pull Back	160	180	167.3913043
B	Front Pin	2	4	4

Multiple Response Prediction				
			99% Confidence Interval	
	Y-hat	S-hat	Lower Bound	Upper Bound
Distance	115.0000	1.9239	109.228	120.772

The predicted capability . . .

Process Capability Analysis Of Confirmation Runs	
	Distance
Upper Spec Limit	121
Lower Spec Limit	109
Mean	115
Standard Deviation	1.923913043
Sigma Capability	4.408
Cpk	1.0395
Cp	1.0395
Defects Per Million	1816.853

The predicted capability for the other setting which would hit the target of 115 inches.

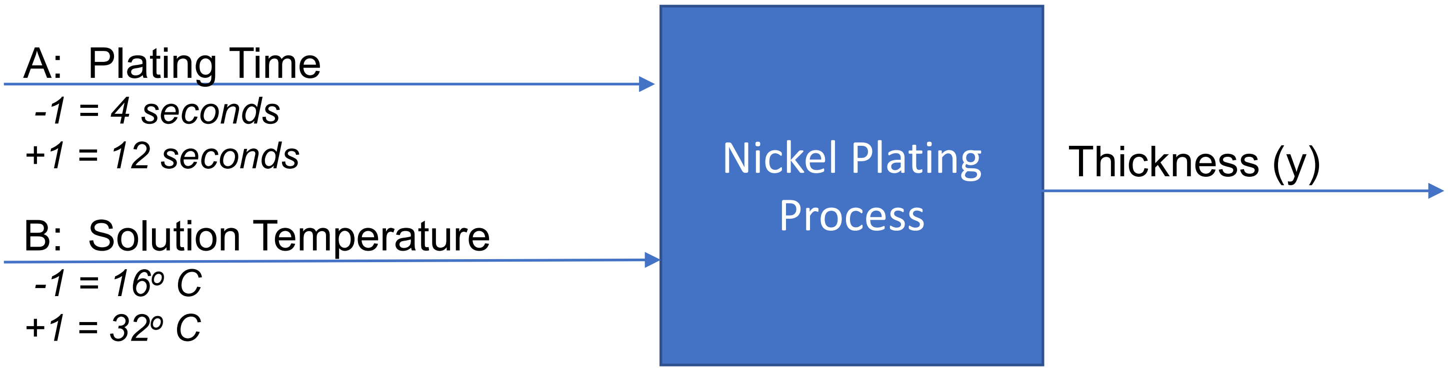
Factor	Name	Low	High	Exper
A	Pull Back	160	180	177.9487179
B	Front Pin	2	4	3

Process Capability Analysis Of Confirmation Runs	
	Distance
Upper Spec Limit	121
Lower Spec Limit	109
Mean	115
Standard Deviation	3.243589744
Sigma Capability	3.019
Cpk	0.6166
Cp	0.6166
Defects Per Million	64342.039

"All models are wrong, but some are useful."
- George Box

Full Factorial Design: Nickel Plating

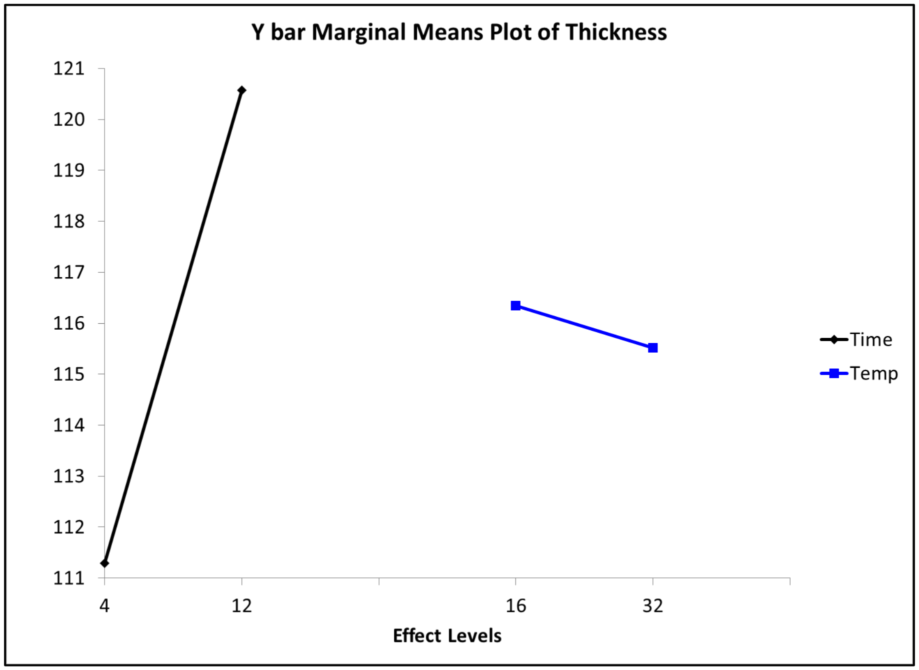
- In a nickel plating process, a team was experiencing issues with plating thickness.
- They set up a full factorial experiment to study the effects of plating time and solution temperature on the resulting plating thickness. The results of their experiment are shown below. The requirement for plating thickness is 110 +/- 5 microns.



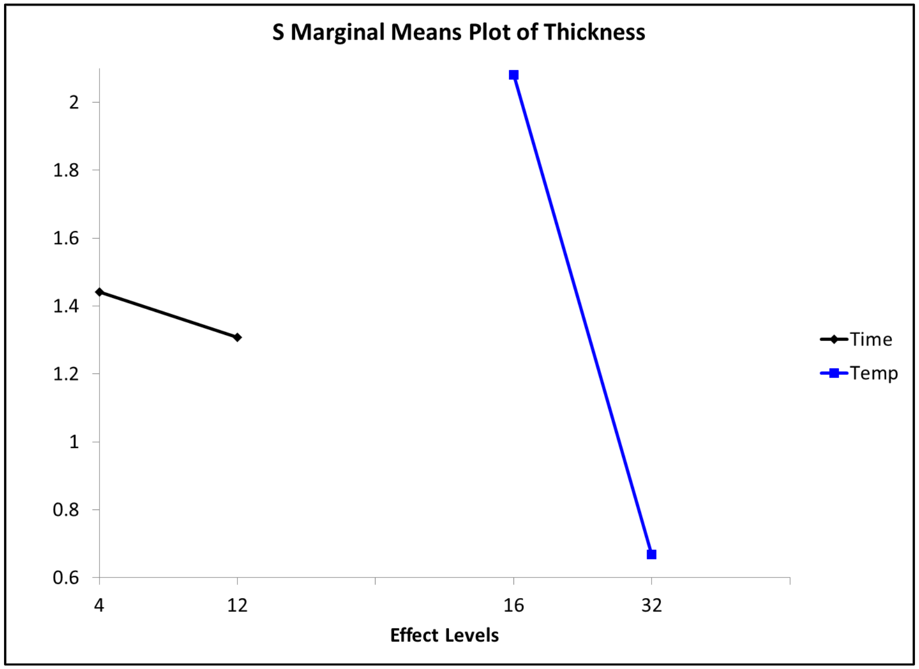
Factor	A	B		Thickness							
Row #	**Time**	**Temp**		**Y1**	**Y2**	**Y3**	**Y4**	**Y5**		**Y bar**	**S**
1	4	16		116.1	116.9	112.6	118.7	114.9		115.84	2.277718
2	4	32		106.7	107.5	105.9	107.1	106.5		106.74	0.60663
3	12	16		116.5	115.5	119.2	114.7	118.3		116.84	1.883614
4	12	32		123.2	125.1	124.5	124	124.7		124.3	0.731437

Data file: DOE 2 factor plating.xlsx

Nickel Plating (Marginal Means Plots)



What variable has the biggest effect on the average plating thickness?



What variable has the biggest effect on the standard deviation?

Nickel Plating (Regression Analysis)

Y-hat Model — Thickness

Factor	Name	Coeff	P(2 Tail)	Tol	Active
Const		115.93	0.0000		
A	Time	4.640	0.0000	1	X
B	Temp	-0.41000	0.2548	1	X
AB		4.140	0.0000	1	X

R^2	0.9527
Adj R^2	0.9438
Std Error	1.5523
F	107.4449
Sig F	0.0000
F_{LOF}	NA
Sig F_{LOF}	NA

Source	SS	df	MS
Regression	776.7	3	258.9
Error	38.6	16	2.4
$Error_{Pure}$	38.6	16	2.4
$Error_{LOF}$	0.0	0	NA
Total	815.3	19	

Factor	Name	Low	High	Exper
A	Time	4	12	8
B	Temp	16	32	24

Multiple Response Prediction

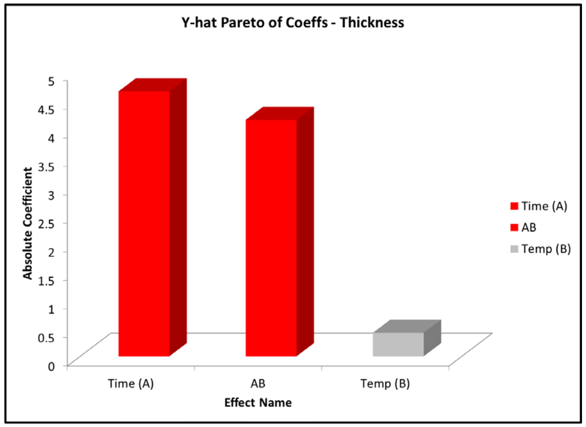
	Y-hat	S-hat	99% Confidence Interval Lower Bound	99% Confidence Interval Upper Bound
Thickness	115.9300	1.3748	111.805	120.055

S-hat Model — Thickness

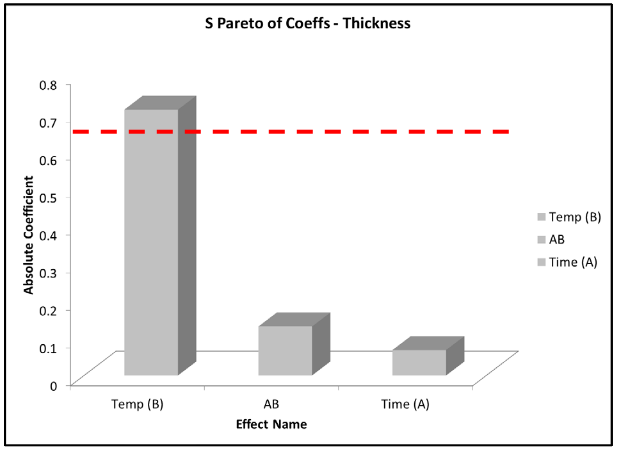
	Factor	Name	Coeff	P(2 Tail)	Tol	Active
	Const		1.375	NA		
C	A	Time	-0.06732	NA	1	X
C	B	Temp	-0.70582	NA	1	X
	AB		0.12973	NA	1	X

R^2	1.0000
Adj R^2	NA
Std Error	NA
F	NA
Sig F	NA
F_{LOF}	NA
Sig F_{LOF}	NA

Source	SS	df	MS
Regression	2.1	3	0.7
Error	0.0	0	NA
$Error_{Pure}$	NA	0	NA
$Error_{LOF}$	NA	0	NA
Total	2.1	3	



Half the constant:
1.375 / 2 = 0.6875



Nickel Plating (Reduced Model)

Y-hat Model

Factor	Name	Thickness Coeff	P(2 Tail)	Tol	Active
Const		115.93	0.0000		
A	Time	4.640	0.0000	1	X
B	Temp	-0.41000	0.2548	1	X
AB		4.140	0.0000	1	X

R^2	0.9527
Adj R^2	0.9438
Std Error	1.5523
F	107.4449
Sig F	0.0000
F_{LOF}	NA
Sig F_{LOF}	NA

Source	SS	df	MS
Regression	776.7	3	258.9
Error	38.6	16	2.4
$Error_{Pure}$	38.6	16	2.4
$Error_{LOF}$	0.0	0	NA
Total	815.3	19	

Factor	Name	Low	High	Exper
A	Time	4	12	8
B	Temp	16	32	24

Multiple Response Prediction

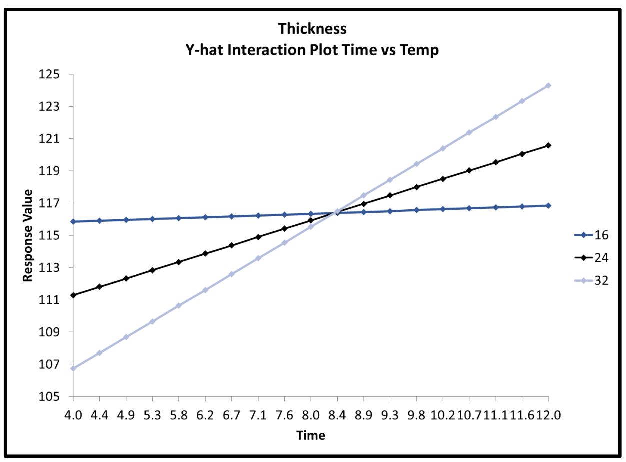
	Y-hat	S-hat	99% Confidence Interval Lower Bound	99% Confidence Interval Upper Bound
Thickness	115.9300	1.3748	111.805	120.055

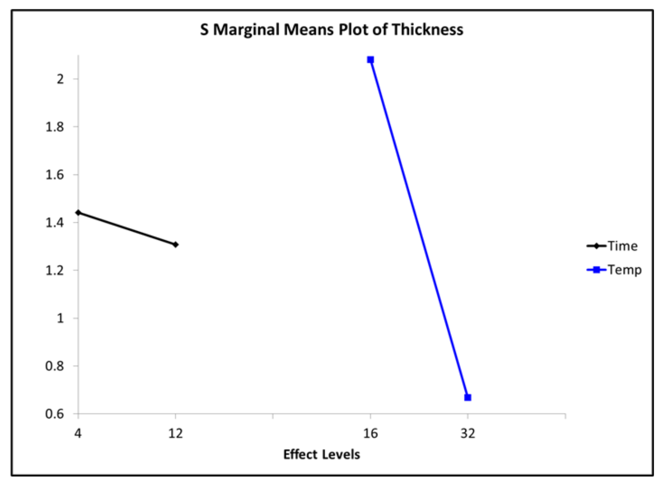
S-hat Model

Factor	Name	Thickness Coeff	P(2 Tail)	Tol	Active
Const		1.375	0.0056		
B	Temp	-0.70582	0.0208	1	X

R^2	0.9589
Adj R^2	0.9383
Std Error	0.2067
F	46.6416
Sig F	0.0208
F_{LOF}	NA
Sig F_{LOF}	NA

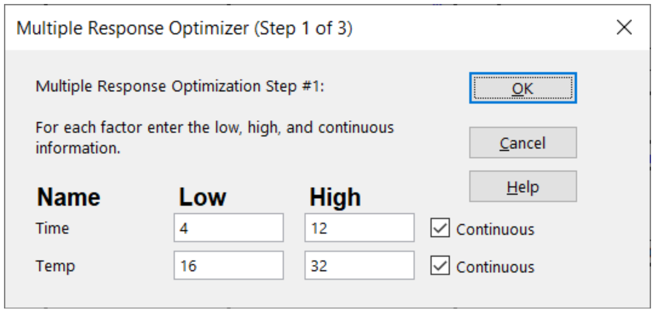
Source	SS	df	MS
Regression	2.0	1	2.0
Error	0.1	2	0.0
$Error_{Pure}$	0.1	2	0.0
$Error_{LOF}$	0.0	0	NA
Total	2.1	3	

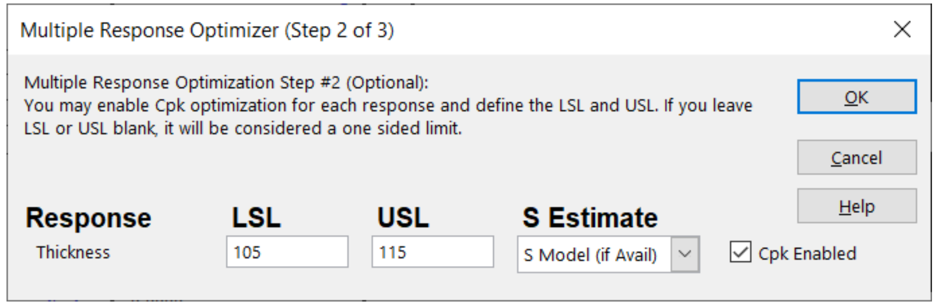




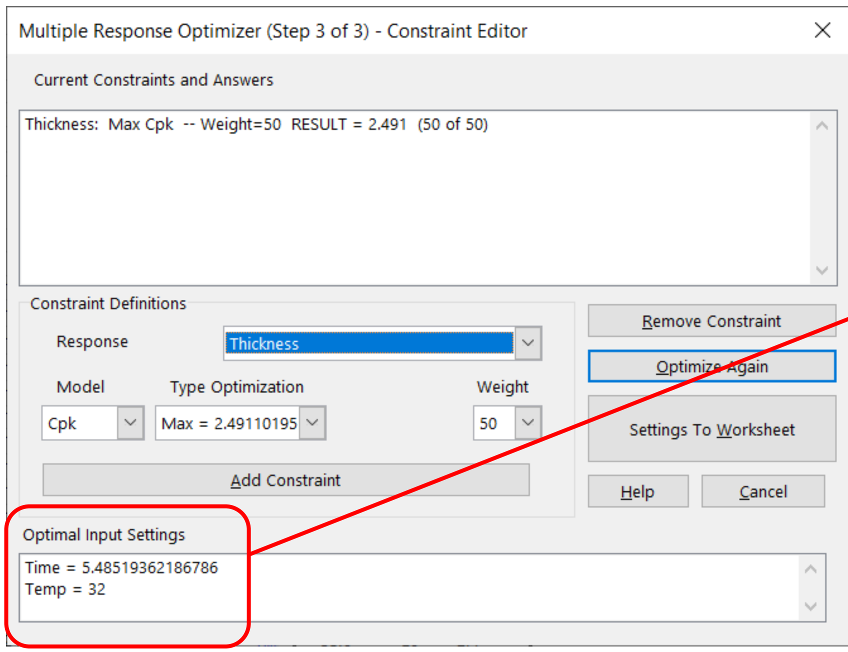
Nickel Plating (Optimization)

- Using the Optimizer in DOE Pro, the following optimal settings were determined:
  - Plating Time (5.5 seconds)
  - Solution Temperature (32° C)





Recall: Specs are 110 +/- 5 microns



Factor	Name	Low	High	Exper
A	Time	4	12	5.485193622
B	Temp	16	32	32

Multiple Response Prediction

			99% Confidence Interval	
	Y-hat	S-hat	Lower Bound	Upper Bound
Thickness	110.0000	0.6690	107.993	112.007

Factor	Name	Low	High	Exper
A	Time	4	12	5.5
B	Temp	16	32	32

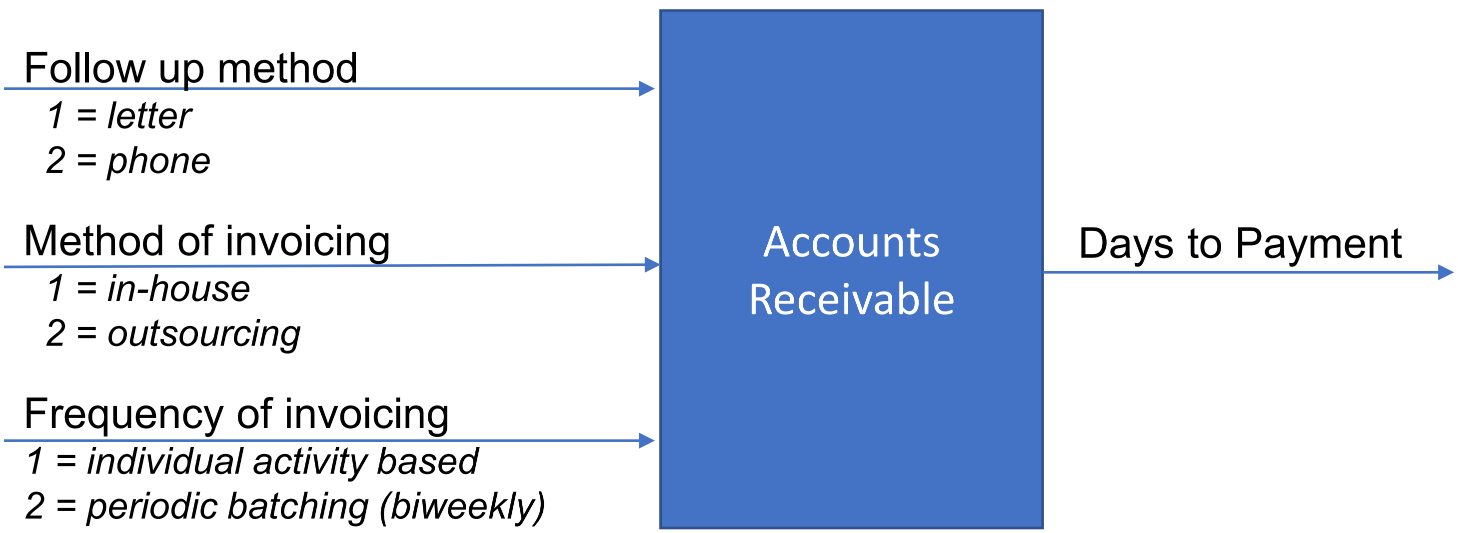
Rounding of Time to 5.5 seconds

Multiple Response Prediction

			99% Confidence Interval	
	Y-hat	S-hat	Lower Bound	Upper Bound
Thickness	110.0325	0.6690	108.025	112.040

Full Factorial Design with 3 Factors

- DOE is applicable any time we are studying how inputs affect one or more outputs. It is not just for manufacturing processes, but applies in the transactional or service industry, design and development, software testing, and beyond
- Consider an accounts receivable process, where cash flow is a major issue. Reducing the time it takes for a customer to pay an invoice is crucial.
- A company found that on average it was taking well over 150 days to get paid, and they had more than $130 million over 30 days past due (and $67 million of that over 60 days past due). Using PF/CE/CNX/SOP, they were able to cut the receivable time almost in half (by 74 days) by removing bottlenecks. For further improvement, they brainstormed other variables affecting payment times and set up an experiment to focus on 3 of these variables as shown.



Accounts Receivable DOE

- They set up the following 8-run full factorial experiment with 6 replications. Data was collected over a 6-month period.

Factor	A	B	C	Days to Pay							
Row #	Follow Up	Method	Frequency	Y1	Y2	Y3	Y4	Y5	Y6	Y bar	S
1	1	1	1	49	46	56	59	47	44	50.167	5.981
2	1	1	2	79	84	86	78	86	91	84.000	4.858
3	1	2	1	51	55	64	53	63	61	57.833	5.529
4	1	2	2	93	96	81	79	80	88	86.167	7.250
5	2	1	1	47	46	44	51	40	49	46.167	3.869
6	2	1	2	59	61	69	62	54	66	61.833	5.269
7	2	2	1	46	52	49	55	59	42	50.500	6.156
8	2	2	2	62	61	64	68	69	60	64.000	3.742

Experimental Trials

	Follow Up Method	Method of Invoicing	Frequency of Invoicing
1	letter	in-house	individual activity based
2	letter	in-house	periodic batching (bi-weekly)
3	letter	outsource	individual activity based
4	letter	outsource	periodic batching (bi-weekly)
5	phone	in-house	individual activity based
6	phone	in-house	periodic batching (bi-weekly)
7	phone	outsource	individual activity based
8	phone	outsource	periodic batching (bi-weekly)

Data file: DOE 3 factor accounts receivable.xlsx

Practice using DOE Pro to Analyze Data

Open the data file and analyze the data. Answer the following questions.

1. Does anything have a significant effect on the days to pay? If so, what? Which of the three main variables has the biggest impact?

2. Does anything have a significant effect on the variation (standard deviation) in payment time? If so, what?

3. Review the marginal means plot for Days to Pay (y-hat) and the interaction plot(s) for any significant interaction(s). What do these suggest about reducing the days to pay?

4. Confirm what the graphs suggest by using the optimizer in DOE Pro. What is the optimal setting for reducing days to pay?

   A: Follow up method ________ (1 = letter, 2 = phone)

   B: Method of invoicing ________ (1 = in-house, 2 = outsourcing)

   C: Frequency of invoicing ________ (1 = individual activity based, 2 = periodic batching)

5. Use the optimal settings to predict the payment time. ________ days

6. If the upper spec limit (goal) is to never exceed 60 days, what is the expect dpm (defects per million) at this new setting? ______________

Data file: DOE 3 factor accounts receivable.xlsx

Accounts Receivable DOE (Regression)

- Initial

Y-hat Model		Days to Pay			
Factor	Name	Coeff	P(2 Tail)	Tol	Active
Const		62.583	0.0000		
A	Follow Up	-6.958	0.0000	1	X
B	Method	2.042	0.0131	1	X
C	Frequency	11.417	0.0000	1	X
AB		-0.41667	0.5989	1	X
AC		-4.125	0.0000	1	X
BC		-0.95833	0.2298	1	X
ABC		0.41667	0.5989	1	X
	R^2	0.8907			
	Adj R^2	0.8715			
	Std Error	5.4444			
	F	46.5464			
	Sig F	0.0000			
	F_{LOF}	NA			
	Sig F_{LOF}	NA			

Factor	Name	Low	High	Exper
A	Follow Up	1	2	1.5
B	Method	1	2	1.5
C	Frequency	1	2	1.5

Multiple Response Prediction				
			99% Confidence Interval	
	Y-hat	S-hat	Lower Bound	Upper Bound
Days to Pay	62.5833	5.3317	46.588	78.578

	S-hat Model		Days to Pay			
	Factor	Name	Coeff	P(2 Tail)	Tol	Active
	Const		5.332	NA		
C	A	Follow Up	-0.57268	NA	1	X
C	B	Method	0.33755	NA	1	X
C	C	Frequency	-0.05186	NA	1	X
	AB		-0.14758	NA	1	X
	AC		-0.20162	NA	1	X
	BC		-0.12141	NA	1	X
	ABC		-0.83244	NA	1	X
		R^2	1.0000			
		Adj R^2	NA			
		Std Error	NA			
		F	NA			
		Sig F	NA			
		F_{LOF}	NA			
		Sig F_{LOF}	NA			

- Reduced model (insignificant terms removed)

Y-hat Model		Days to Pay			
Factor	Name	Coeff	P(2 Tail)	Tol	Active
Const		62.583	0.0000		
A	Follow Up	-6.958	0.0000	1	X
B	Method	2.042	0.0119	1	X
C	Frequency	11.417	0.0000	1	X
AC		-4.125	0.0000	1	X
	R^2	0.8851			
	Adj R^2	0.8744			
	Std Error	5.3839			
	F	82.7736			
	Sig F	0.0000			
	F_{LOF}	0.6832			
	Sig F_{LOF}	0.5676			

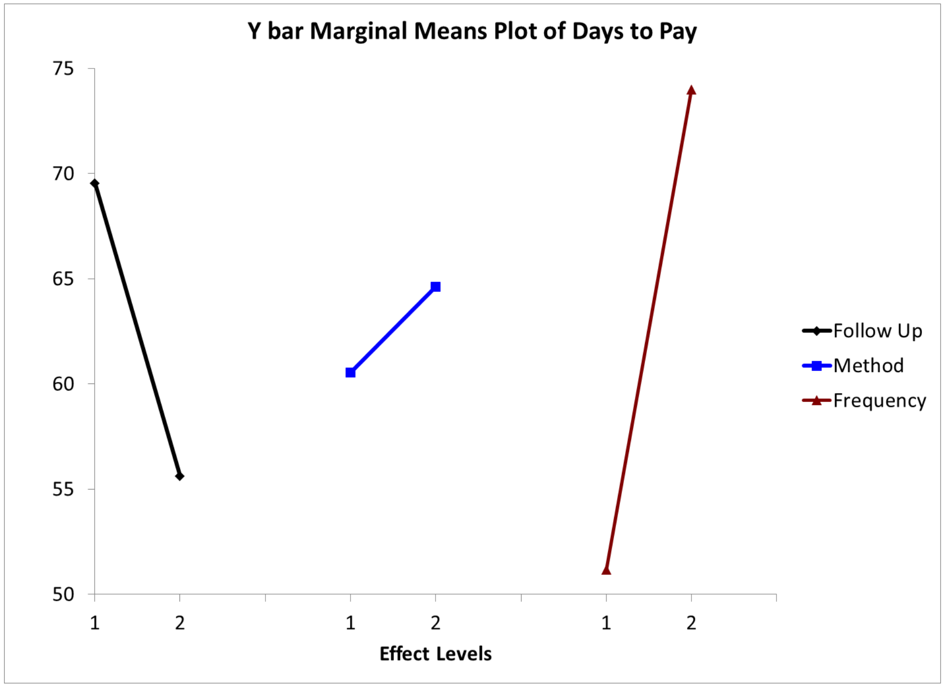
Factor	Name	Low	High	Exper
A	Follow Up	1	2	1.5
B	Method	1	2	1.5
C	Frequency	1	2	1.5

Multiple Response Prediction				
			99% Confidence Interval	
	Y-hat	S-hat	Lower Bound	Upper Bound
Days to Pay	62.5833	5.3317	46.588	78.578

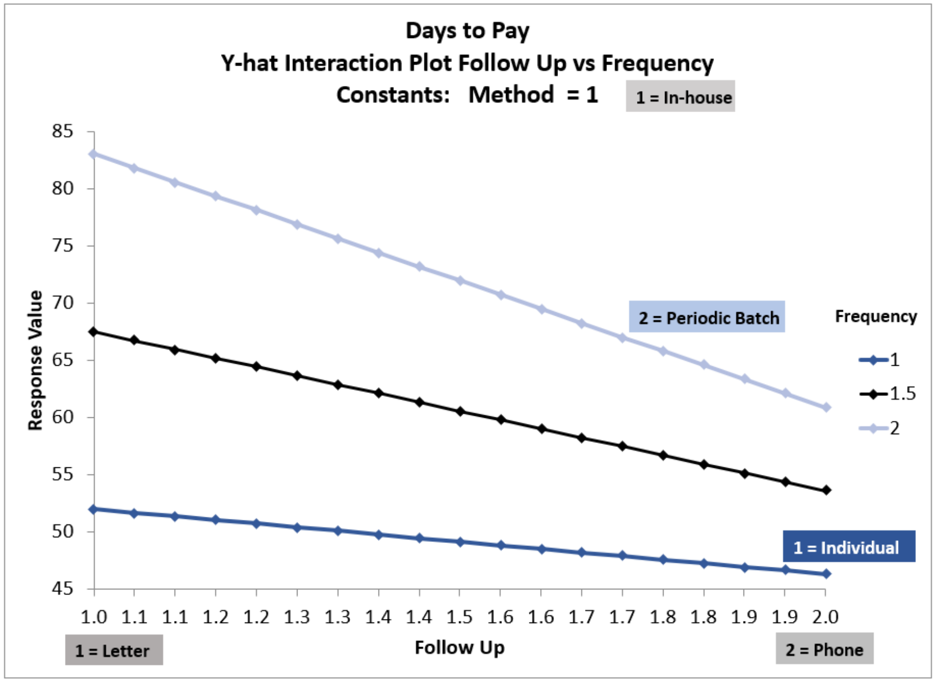
S-hat Model		Days to Pay			
Factor	Name	Coeff	P(2 Tail)	Tol	Active
Const		5.332	0.0000		
	R^2	0.0000			
	Adj R^2	0.0000			
	Std Error	1.1782			
	F	NA			
	Sig F	NA			
	F_{LOF}	NA			
	Sig F_{LOF}	NA			

Accounts Receivable DOE (Regression) (cont.)

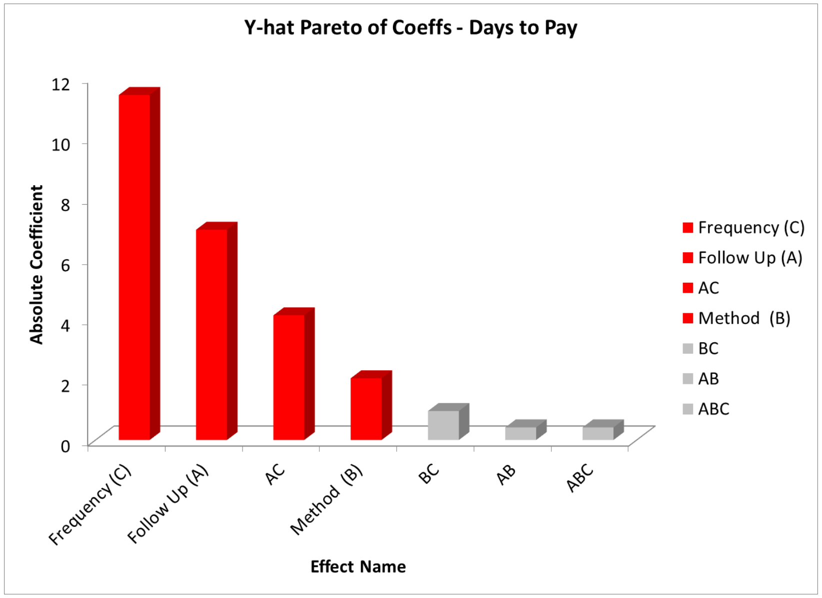
- Marginal Means Plots



Interaction Plot (AC) (Follow-up x Frequency)



- Pareto of Regression Coefficients



Optimized Settings

- Follow-Up = 2 (by phone)
- Method of Invoicing = 1 (in-house)
- Frequency of Invoicing = 1 (individual activity based)

Factor	Name	Low	High	Exper
A	Follow Up	1	2	2
B	Method	1	2	1
C	Frequency	1	2	1

- Predicted Days to Pay = 46.29

Multiple Response Prediction				
			99% Confidence Interval	
	Y-hat	S-hat	Lower Bound	Upper Bound
Days to Pay	46.2917	5.3317	30.297	62.287

- Assuming an upper spec limit of 60 days, the predicted capability shows a defects per million (dpm) = 5,068 (so we'd expect to exceed the 60 days only about 0.5% of the time!
- Of course, confirmation is critical! The company did further pilot testing and the results were validated.
- The company's finance department has shown that this reduction in time until payment equates to more than $450,000 savings annually due to improved cash flow.

Process Capability Analysis Of Confirmation Runs	
	Days to Pay
Upper Spec Limit	60
Lower Spec Limit	
Mean	46.29166667
Standard Deviation	5.331693566
Sigma Capability	4.071
Cpk	0.8570
Cp	NA
Defects Per Million	5068.759

Case Study: Full Factorial Design with 4 Factors

(Understanding Industrial Designed Experiments, Pages 8-218 through 8-228)

- In a precision machining process, a machined lathe part used in a critical assembly had to meet a circular runout tolerance of 500 micro inches. A company was having issues meeting this tolerance, averaging 550 micro inches.
- The team brainstormed possible contributors to excessive runout. The factors were related to the tool cutting pressure and the method of holding the part during machining. After further brainstorming factors that could affect the tool cutting pressure and method of holding the part, they identified the following 4 factors which were varied using a 16-run full factorial design with 3 replicates:

	Low	High
– A: Finish Cut	yes (1)	no (2)
– B: Jaws (geometry)	old style (1)	new style (2)
– C: Chuck Pressure	20 psi	35 psi
– D: Feed Rate	0.001"/Rev	0.003"/Rev

- The goal of the experiment was to answer the following questions:
  - Which factors, if any, have a significant effect on the runout?
  - Which factors, if any, have a significant effect on the variability?
  - What are the best settings to minimize both runout and variation, while ideally keeping the feed rate high? (USL for runout = 500)

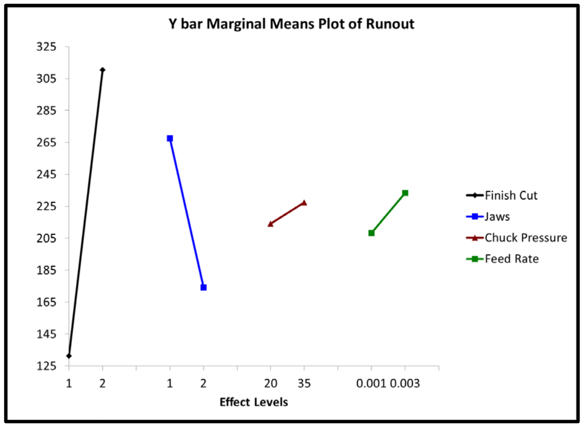
4 Factor Full Factorial (Circular Runout)

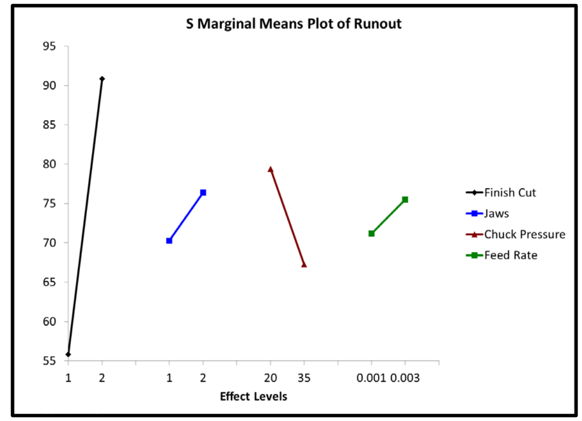
- The experiment and resulting data:

Factor	A	B	C	D		Runout					
Row #	Finish Cut	Jaws	Chuck Pressure	Feed Rate		Y1	Y2	Y3		Y bar	S
1	1	1	20	0.001		50	300	170		173.3333	125.0333
2	1	1	20	0.003		180	220	100		166.6667	61.10101
3	1	1	35	0.001		140	180	300		206.6667	83.26664
4	1	1	35	0.003		140	200	200		180	34.64102
5	1	2	20	0.001		40	60	20		40	20
6	1	2	20	0.003		200	50	100		116.6667	76.37626
7	1	2	35	0.001		120	60	60		80	34.64102
8	1	2	35	0.003		100	80	80		86.66667	11.54701
9	2	1	20	0.001		280	400	450		376.6667	87.36895
10	2	1	20	0.003		320	390	390		366.6667	40.41452
11	2	1	35	0.001		220	290	300		270	43.58899
12	2	1	35	0.003		500	350	350		400	86.60254
13	2	2	20	0.001		180	260	340		260	80
14	2	2	20	0.003		120	380	140		213.3333	144.6836
15	2	2	35	0.001		200	370	210		260	95.39392
16	2	2	35	0.003		300	500	210		336.6667	148.4363

Data file: DOE 4 factor circular runout.xlsx

4 Factor Full Factorial (Circular Runout) (Initial Regression)





Y-hat Model

Factor	Name	Runout Coeff	P(2 Tail)	Tol	Active
Const		220.83	0.0000		
A	Finish Cut	89.583	0.0000	1	X
B	Jaws	-46.667	0.0005	1	X
C	Chuck Pressure	6.667	0.5851	x	X
D	Feed Rate	12.500	0.3088	1	X
AB		3.750	0.7584	1	X
AC		-0.41667	0.9727	1	X
AD		6.250	0.6087	1	X
BC		10.000	0.4142	1	X
BD		1.667	0.8912	1	X
CD		10.833	0.3768	1	X
ABC		14.583	0.2365	1	X
ABD		-12.917	0.2932	1	X
ACD		22.083	0.0770	1	X
BCD		-4.167	0.7326	1	X
ABCD		2.083	0.8642	1	X

R^2	0.7121
Adj R^2	0.5772
Std Error	83.7407
F	5.2779
Sig F	0.0000
F_{LOF}	NA
Sig F_{LOF}	NA

Factor	Name	Low	High	Exper
A	Finish Cut	1	2	1.5
B	Jaws	1	2	1.5
C	Chuck Pressure	20	35	27.5
D	Feed Rate	0.001	0.003	0.002

Multiple Response Prediction

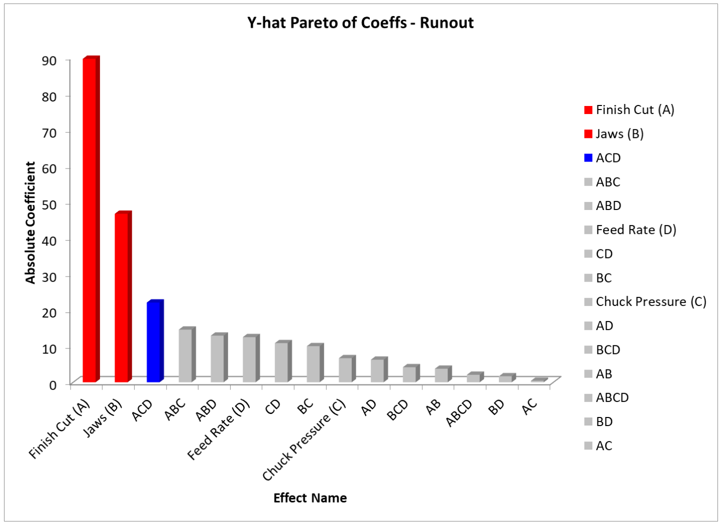
	Y-hat	S-hat	99% Confidence Interval Lower Bound	Upper Bound
Runout	220.8333	73.3184	0.878	440.789

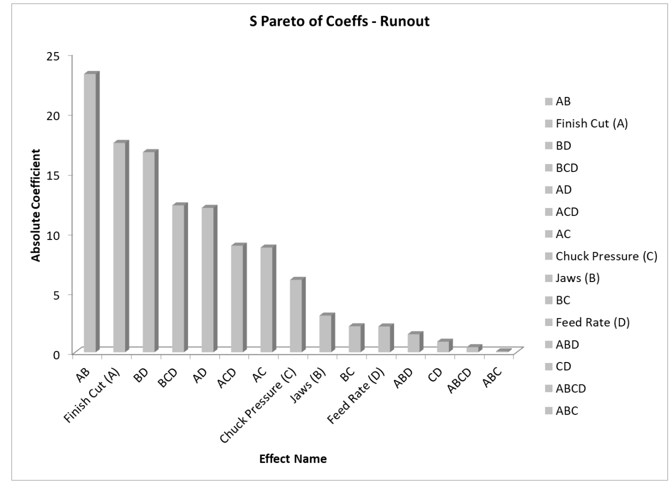
S-hat Model

Factor	Name	Runout Coeff	P(2 Tail)	Tol	Active
Const		73.318	NA		
A	Finish Cut	17.493	NA	1	X
B	Jaws	3.066	NA	1	X
C	Chuck Pressure	-6.054	NA	1	X
D	Feed Rate	2.157	NA	1	X
AB		23.251	NA	1	X
AC		8.748	NA	1	X
AD		12.066	NA	1	X
BC		2.174	NA	1	X
BD		16.719	NA	1	X
CD		0.88520	NA	1	X
ABC		-0.08124	NA	1	X
ABD		-1.511	NA	1	X
ACD		8.906	NA	1	X
BCD		-12.274	NA	1	X
ABCD		-0.42701	NA	1	X

R^2	1.0000
Adj R^2	NA
Std Error	NA
F	NA
Sig F	NA
F_{LOF}	NA
Sig F_{LOF}	NA

4 Factor Full Factorial (Circular Runout) (Reduced Regression)





Y-hat Model

Factor	Name	Runout Coeff	P(2 Tail)	Tol	Active
Const		220.83	0.0000		
A	Finish Cut	89.583	0.0000	1	X
B	Jaws	-46.667	0.0002	1	X

R^2	0.6282
Adj R^2	0.6117
Std Error	80.2531
F	38.0201
Sig F	0.0000
F_{LOF}	0.1027
Sig F_{LOF}	0.7501

Factor	Name	Low	High	Exper
A	Finish Cut	1	2	1.5
B	Jaws	1	2	1.5
C	Chuck Pressure	20	35	27.5
D	Feed Rate	0.001	0.003	0.002

Multiple Response Prediction

	Y-hat	S-hat	99% Confidence Interval Lower Bound	99% Confidence Interval Upper Bound
Runout	220.8333	73.3184	0.878	440.789

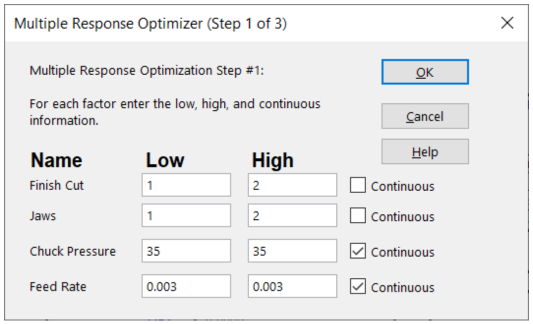
S-hat Model

Factor	Name	Runout Coeff	P(2 Tail)	Tol	Active
Const		73.318	0.0000		

R^2	0.0000
Adj R^2	0.0000
Std Error	41.7856
F	NA
Sig F	NA
F_{LOF}	NA
Sig F_{LOF}	NA

4 Factor Full Factorial (Optimization and Confirmation)

- Optimization was set to minimize y-hat (since nothing was significant for s-hat)
- Constraints: Chuck pressure and feed rate were preferred to be kept at their current settings (35 psi and 0.003”/rev)



Factor	Name	Low	High	Exper
A	Finish Cut	1	2	1
B	Jaws	1	2	2
C	Chuck Pressure	20	35	35
D	Feed Rate	0.001	0.003	0.003

Multiple Response Prediction				
			99% Confidence Interval	
	Y-hat	S-hat	Lower Bound	Upper Bound
Runout	84.5833	73.3184	-135.372	304.539

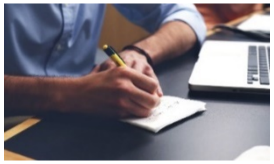
Optimized settings used for confirmation:

A: Finish Cut	yes (1)
B: Jaws (geometry)	new style (2)
C: Chuck Pressure	35 psi
D: Feed Rate	0.003”/Rev

- The predicted circular runout was 84.58 micro inches, and the predicted standard deviation was 73.3 micro inches

- A confirmation run of 18 parts was conducted with the finish cut added and the new style jaws (chuck pressure was held at 35psi and the feed rate was held at 0.003” per revolution). The confirmation run yielded an average runout of 66 micro inches with a standard deviation of 48 micro inches, both in reasonable agreement with the predicted values (and slightly better!).

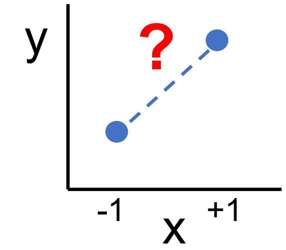
Key Takeaways



- As a review, you may want to pause the video at this point and summarize the key learnings from this session, at least from a high-level view. When you are finished, resume the video.

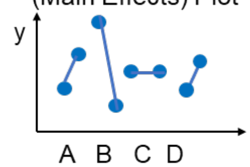
Key Takeaways

- Full factorial designs test all possible combinations
- Full factorial designs are orthogonal (balanced), which allows us to independently estimate effects
- Testing at two levels produces a linear model; confirmation at or near the center is critical to validate that assumption

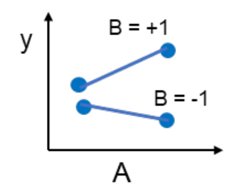


- Full factorial designs allow us to study all possible interactions
- An interaction between two factors means the effect that one factor is having on the response, depends on the setting of another. It is a "combination" effect.
- When analyzing the results from a DOE, always look at both the graphical and statistical analysis
- Helpful graphs include:
  - Marginal means (main effects) plots
  - Pareto of regression coefficients
  - Interaction, surface, and/or contour plots

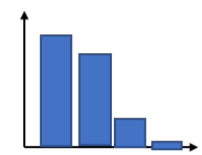
Marginal Means (Main Effects) Plot



Interaction Plot

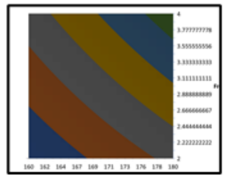


Pareto Plot

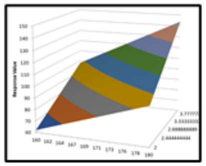


- Statistical analysis (regression) items to review:
  - p-values (what should be included in the prediction model?)
  - R^2 and adjusted R^2 of the model

Contour Plot



Surface Plot



Key Takeaways (cont.)

- When using software such as DOE Pro, the analysis is done in coded units
  - Low setting (-1)   High setting (+1)
  - DOE Pro automatically codes and uncodes for us
- Coding improves orthogonality when estimating interaction effects, and allows us to compare the size of coefficients since everything is on the same scale
- When removing terms from a model
  - Remember the rule of hierarchy: If an interaction or higher order term in significant, keep all main effects involved in that term in the model
  - Re-run the regression to update the model
- When optimizing, don't forget to consider s-hat and how you can reduce variation (DOE Pro will allow for Cpk optimization, which considers both the mean and standard deviation)
- Always run confirmation (validation) tests. As George Box said: "All models are wrong, but some are useful." The useful ones are the ones that confirm!

Supplemental Material

- Suggested Reading:
  - ***Lean Six Sigma: A Tools Guide*** by Adams, Kiemele, Pollock and Quan (pp. 139-146)
  - ***Basic Statistics – Tools for Continuous Improvement*** by Kiemele, Schmidt and Berdine, 4th edition (pp. 8-9 – 8-27)
  - ***Design for Six Sigma: The Tool Guide for Practitioners*** by Reagan and Kiemele (pp. 79-100)
  - ***Understanding Industrial Designed Experiments*** by Schmidt and Launsby, 4th edition (pp. 2-1 – 2-57, 3-1 – 3-9)
  - Air Academy's app: ***Six Sigma Quick Tools***

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- SPC XL™ software training tutorials:
  - https://airacad.com/our-insights/training-videos/spc-xl/

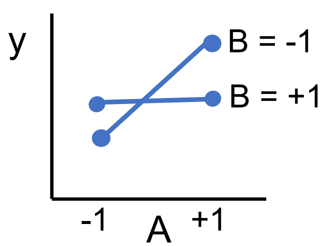
- The data files for this session can be downloaded from the site where you are accessing this course

Additional Practice / Review Questions

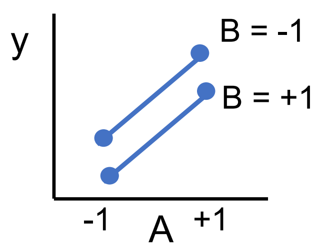
1) In a full factorial design, if we want to test 3 factors at two settings for each, how many total runs (or combinations) will there be to test?

2) Which of the following pictures indicates an interaction between factors A and B? (select all that apply)

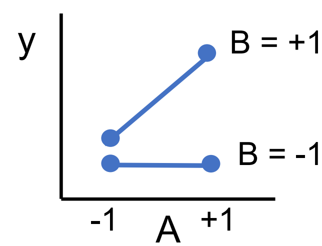
a.



b.



c.



3) In picture a above, if the goal is to maximize y, what is the best setting for factors A and B?

4) When using p-values, what is the rule of thumb for determining significance and deciding which terms to include in the model?

5) In a 2-level design, if there are no p-values when analyzing s-hat, what is the approximate rule of thumb for determining significance and deciding which terms to include in the model?

Additional Practice / Review Questions

6) Open the data file: DOE practice data.xlsx. Two factors, A and B, were tested to determine their effect on part thickness. The requirement for thickness is 53 +/- 7.

A	B	C	D	E	F	G	H	I	J
Factor	A	B		Thickness					
Row #	A	B		Y1	Y2	Y3		Y bar	S
1	5	70		39	47	36		40.66667	5.686241
2	5	90		65	60	54		59.66667	5.507571
3	10	70		78	75	76		76.33333	1.527525
4	10	90		44	46	48		46	2

Data file: DOE practice data.xlsx

a) What are the significant effects for the average part thickness (y-hat)?

b) What are the significant effects for the standard deviation in part thickness (s-hat)?

c) Assume factors A and B can take on any value, on a continuous scale, within the ranges tested. Recommend the best settings for A and B to optimize part thickness. (the requirement for thickness is 53 +/- 7)

d) Using your recommended settings, what are the predicted values for:

part thickness: ____________ standard deviation: ____________

e) What is predicted Cp, Cpk, and defects per million (dpm) using these recommended settings? Cp = _________ Cpk = __________ dpm = ____________

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