Three Level Designs

User Agreement and Copyright Information

Three Level Designs

In this session, we will discuss:

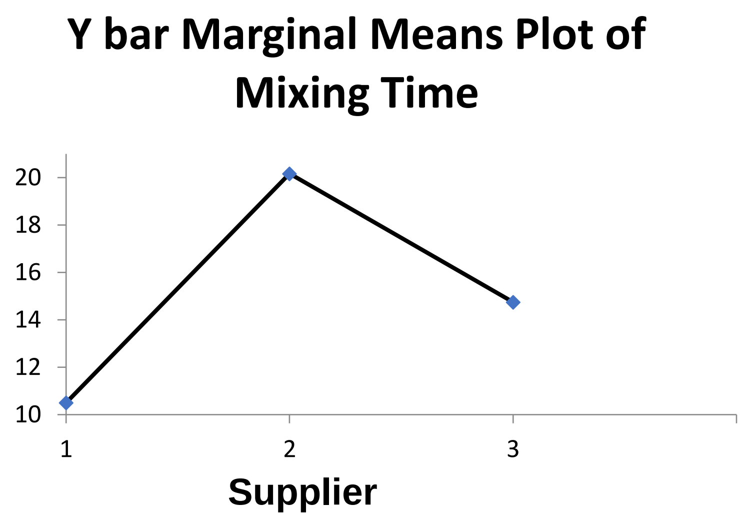
- Types of Input Factors: Qualitative versus Quantitative
- KISS Guidelines Flowchart
- Three Level Designs
  - Full Factorial (3^k Designs)
  - L_{18} Screening Designs
  - Response Surface Modeling Designs
    - Box – Behnken Design
    - Central Composite Design (CCD)

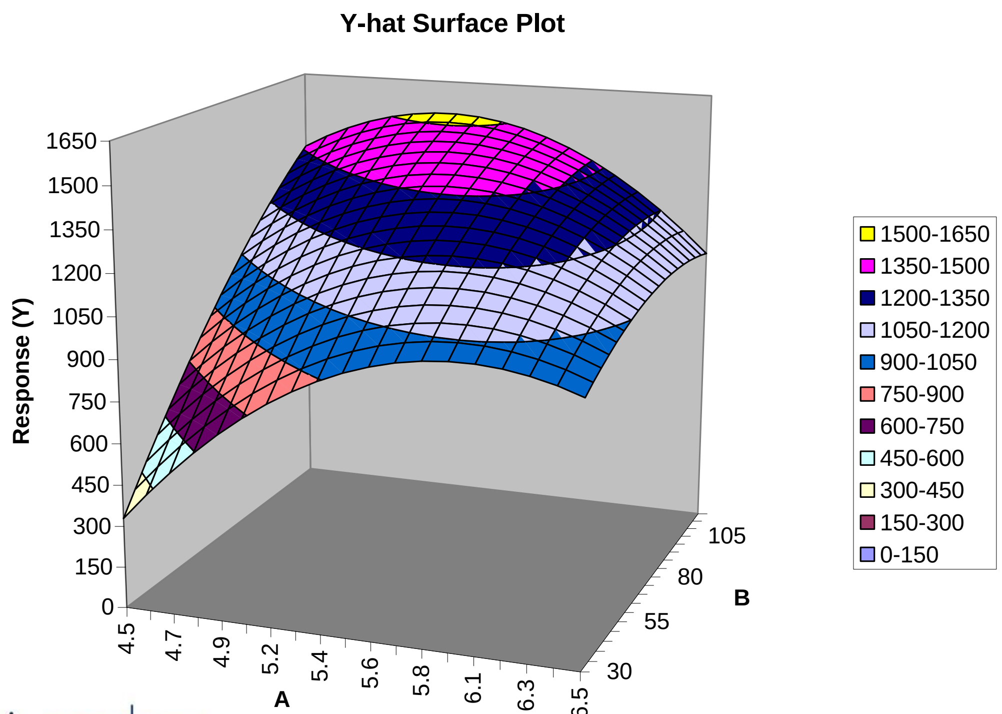
Take Note

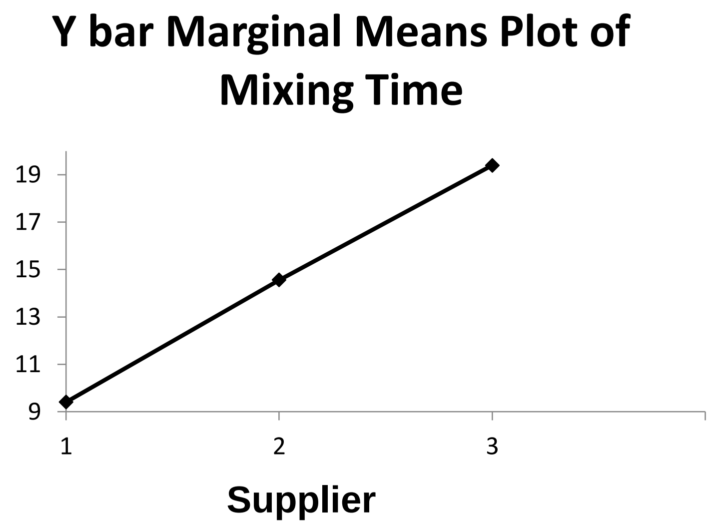
- A list of supplemental material and additional practice/review questions for this session are provided at the end of this presentation
- You can download the pdf of this presentation, along with any supporting data files, on the site where you are accessing this course

Input Factor Types and Levels

- Types of Input Factors
  - Qualitative (Input factor significance)
  - Quantitative (Models!)
  - Mixed factors (Both)
- Number of levels for each input factor
  - Two levels
  - Three levels
  - Mixed levels



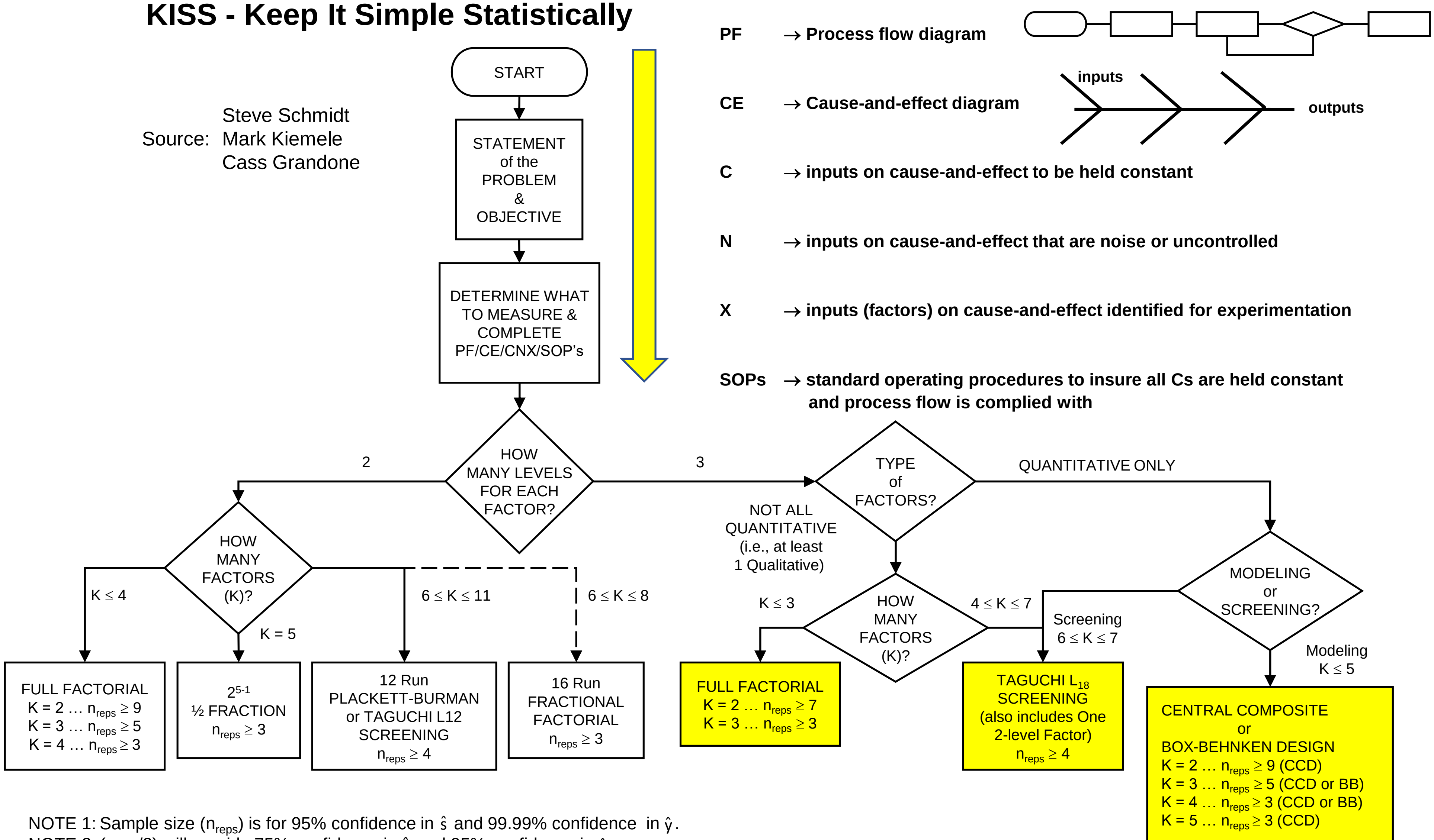




KISS Guidelines for Choosing an Experimental Design

KISS - Keep It Simple Statistically

Source: Steve Schmidt
Mark Kiemele
Cass Grandone

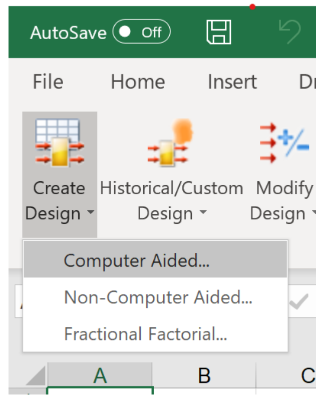


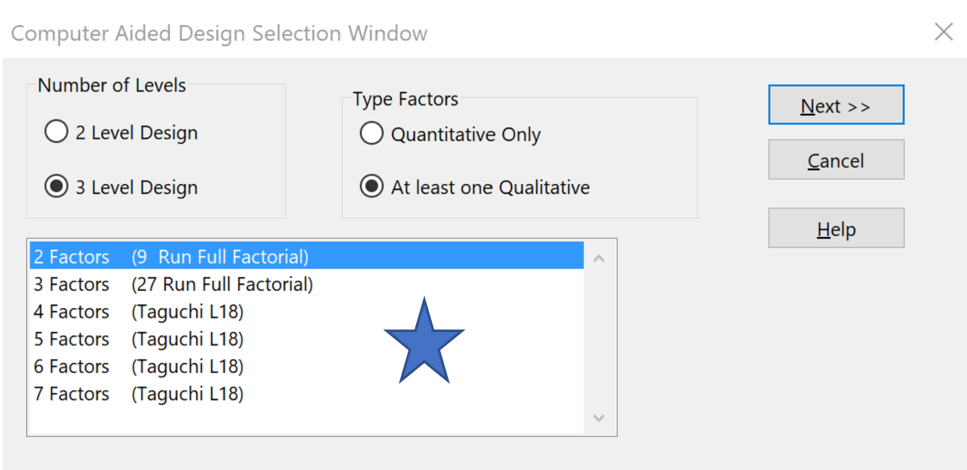
NOTE 1: Sample size (n_{reps}) is for 95% confidence in $\hat{s}$ and 99.99% confidence in $\hat{y}$.
NOTE 2: (n_{reps}/2) will provide 75% confidence in $\hat{s}$ and 95% confidence in $\hat{y}$.
NOTE 3: The 12 Run Plackett-Burman or L12 is very sensitive to large numbers of interactions. If this is the case, you would be better off using the 16 Run Fractional Factorial or a smaller number of variables in 2 or more full factorial experiments.
NOTE 4: For more complete 2-level design options, see next page.

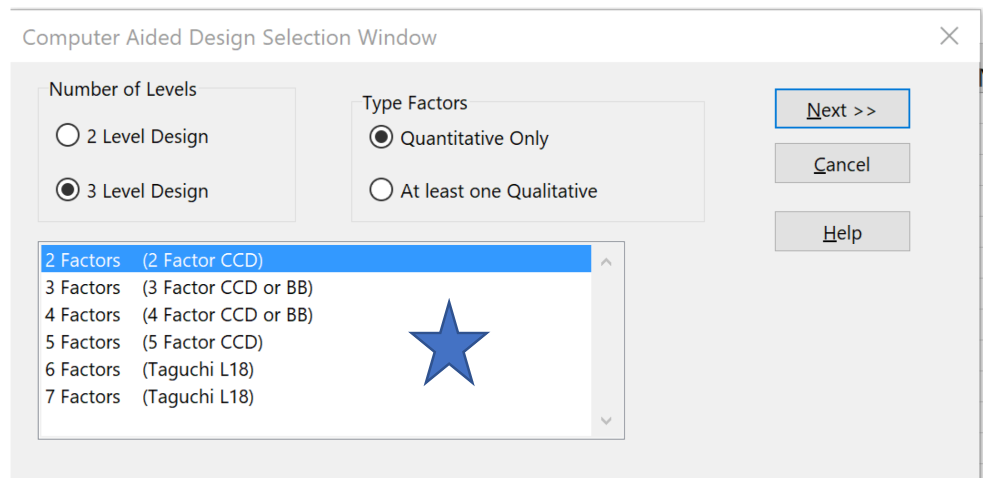
DOE PRO XL Three Level Designs

DOE PRO XL follows the KISS Guidelines!

- **<u>DOE PRO XL > Create Design > Computer Aided …</u>**







KISS Guidelines for Choosing and Experimental Design

KISS - Keep It Simple Statistically

Source: Steve Schmidt
Mark Kiemele
Cass Grandone

PF → **Process flow diagram**

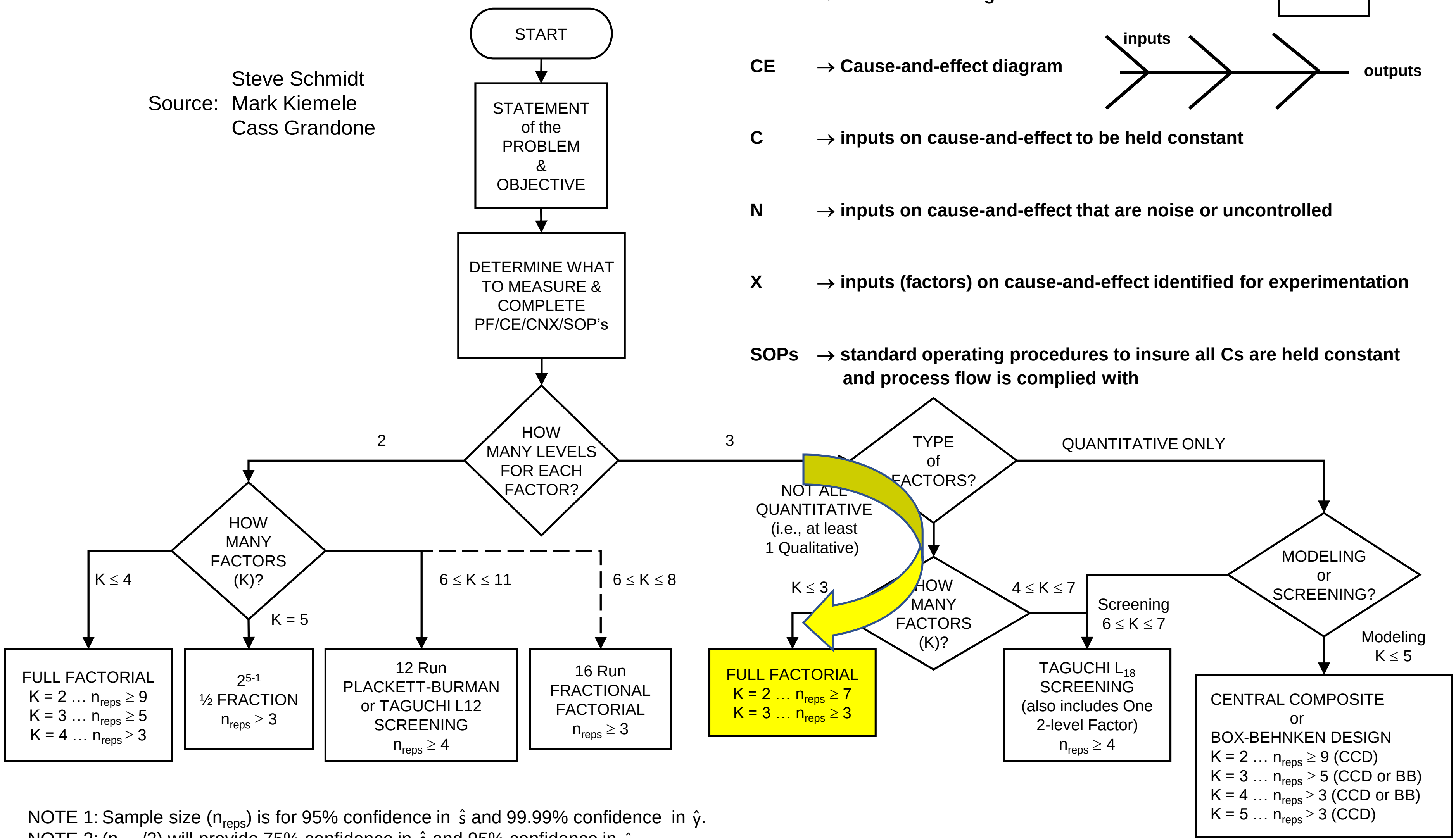
CE → **Cause-and-effect diagram**

C → **inputs on cause-and-effect to be held constant**

N → **inputs on cause-and-effect that are noise or uncontrolled**

X → **inputs (factors) on cause-and-effect identified for experimentation**

SOPs → **standard operating procedures to insure all Cs are held constant and process flow is complied with**



NOTE 1: Sample size (n_{reps}) is for 95% confidence in $\hat{s}$ and 99.99% confidence in $\hat{y}$.
NOTE 2: ($n_{reps}/2$) will provide 75% confidence in $\hat{s}$ and 95% confidence in $\hat{y}$.
NOTE 3: The 12 Run Plackett-Burman or L12 is very sensitive to large numbers of interactions. If this is the case, you would be better off using the 16 Run Fractional Factorial or a smaller number of variables in 2 or more full factorial experiments.
NOTE 4: For more complete 2-level design options, see next page.

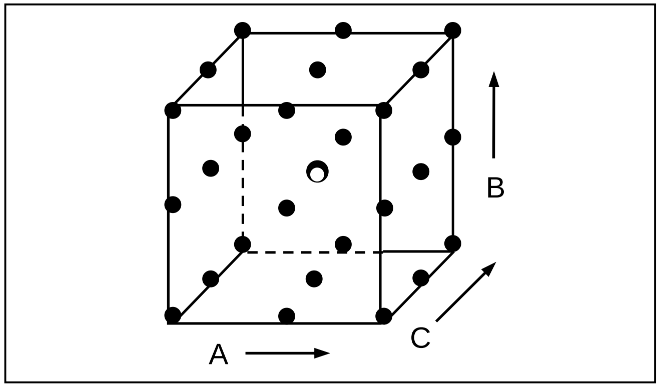
Three Level Full Factorial Designs

OBJECTIVE: To test all possible combinations ($n = 3^k$)

ADVANTAGES: Can estimate all mains, all quadratics, and all linear interactions. Can mix qualitative and quantitative factors.

DISADVANTAGES: Very costly when k > 3. Very inefficient due to sparsity of high-order interactions.

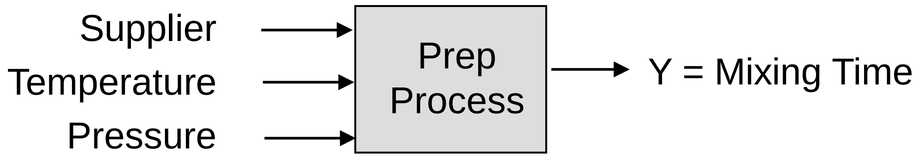
Full Factorial Design Space



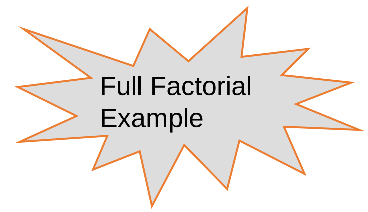
27 Full Factorial Design Conditions for K=3

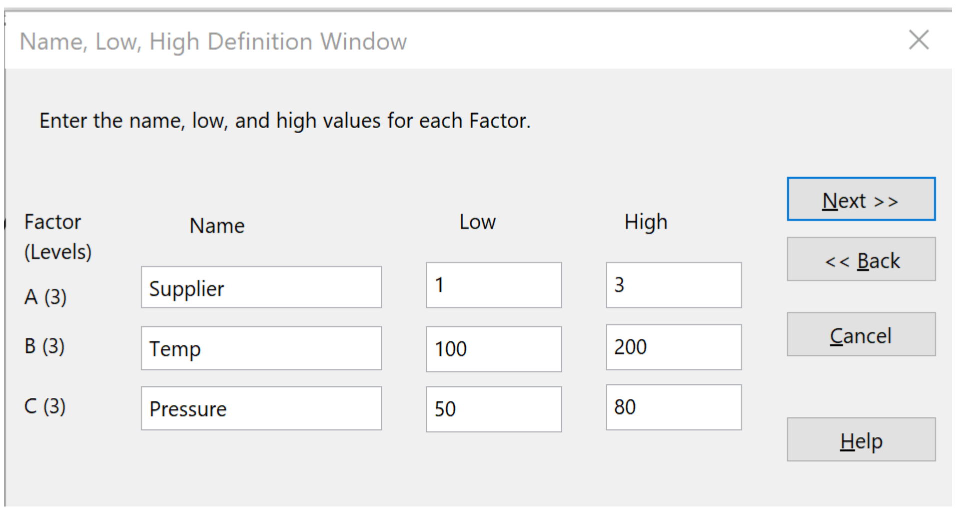
	Factors				Factors				Factors		
Run	**A**	**B**	**C**	**Run**	**A**	**B**	**C**	**Run**	**A**	**B**	**C**
1	1	1	1	10	0	1	1	19	-1	1	1
2	1	1	0	11	0	1	0	20	-1	1	0
3	1	1	-1	12	0	1	-1	21	-1	1	-1
4	1	0	1	13	0	0	1	22	-1	0	1
5	1	0	0	14	0	0	0	23	-1	0	0
6	1	0	-1	15	0	0	-1	24	-1	0	-1
7	1	-1	1	16	0	-1	1	25	-1	-1	1
8	1	-1	0	17	0	-1	0	26	-1	-1	0
9	1	-1	-1	18	0	-1	-1	27	-1	-1	-1

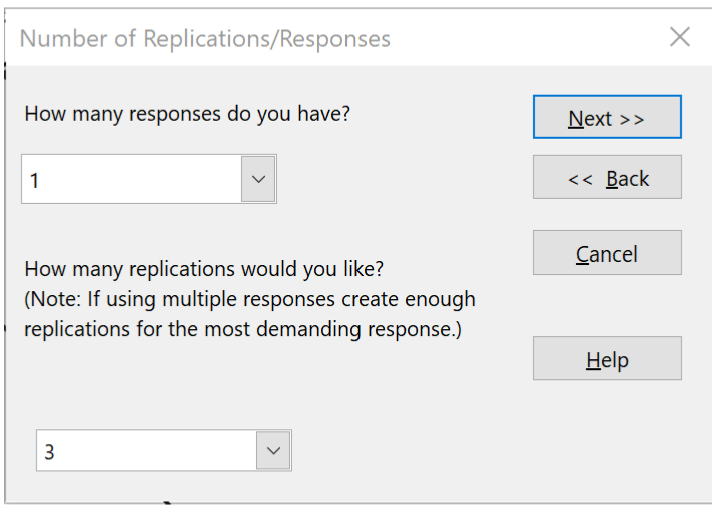
DOE PRO XL Full Factorial Design Example



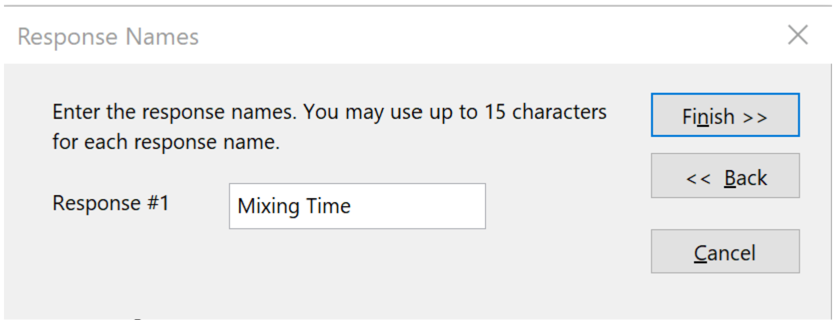
Three Level Designs – Data Files







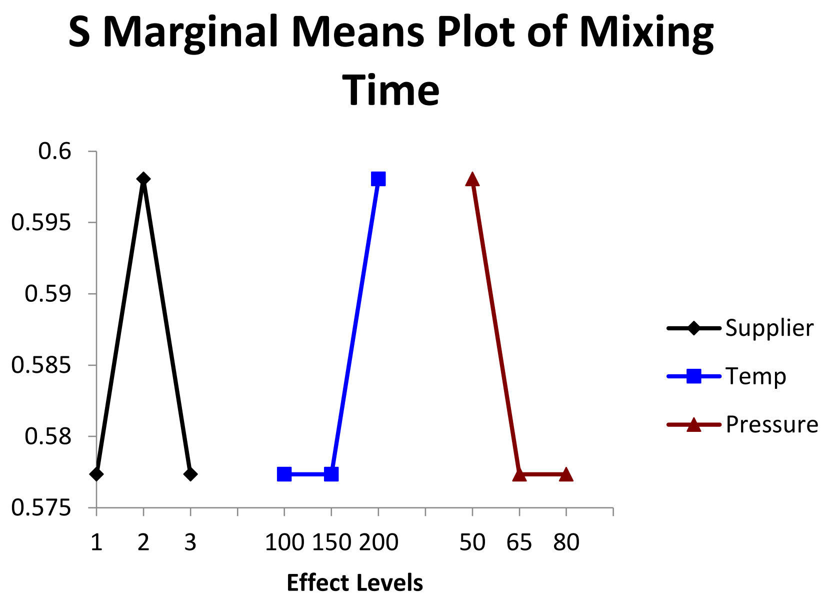
DOE PRO XL Full Factorial Design Example (cont.)

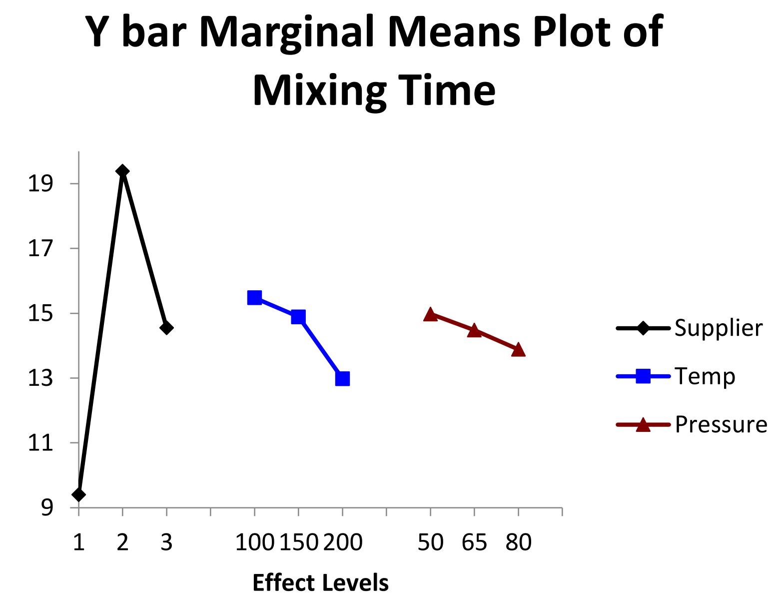


Factor	A	B	C		Mixing Time		
Row #	Supplier	Temp	Pressure		Y1	Y2	Y3
1	3	200	80		12	12	13
2	3	200	65		13	12	13
3	3	200	50		14	13	13
4	3	150	80		15	15	16
5	3	150	65		15	15	16
6	3	150	50		16	16	15
7	3	100	80		15	15	16
8	3	100	65		15	15	16
9	3	100	50		16	16	15
10	2	200	80		17	17	18
11	2	200	65		18	18	17
12	2	200	50		19	19.5	18
13	2	150	80		18	19	18
14	2	150	65		20	21	20
15	2	150	50		20	20	21
16	2	100	80		20	19	20
17	2	100	65		20	21	21
18	2	100	50		21	21	22
19	1	200	80		8	8	7
20	1	200	65		9	8	8
21	1	200	50		9	9	8
22	1	150	80		10	9	9
23	1	150	65		10	9	10
24	1	150	50		10	10	9
25	1	100	80		9	10	10
26	1	100	65		10	11	10
27	1	100	50		11	11	12

DOE PRO XL Full Factorial Design Example (cont.)

DOE PRO XL > Analyze Design > Marginal Means Plot…





DOE PRO XL Full Factorial Design Example (cont.)

DOE PRO XL > Analyze Design > Multiple Response Regression

Y-hat Model

Factor	Name	Coeff (Mixing Time)	P(2 Tail)	Tol	Active
Const		19.858	0.0000		
A	Supplier	2.870	0.0000	0.2000	X
B	Temp	-1.333	0.0000	0.2000	X
C	Pressure	-0.75926	0.0001	0.2000	X
AB		-0.11111	0.2658	1	
AC		0.11111	0.2658	1	
BC		0.01389	0.8888	1	
ABC		-0.16667	0.1739	1	
AA		-7.407	0.0000	1	X
BB		-0.65741	0.0000	1	X
CC		-0.04630	0.7419	1	
AAB		0.08333	0.6285	0.3333	
ABB		-0.55556	0.0019	0.3333	
AAC		0.47222	0.0076	0.3333	
ACC		0.11111	0.5192	0.3333	
BBC		-0.15278	0.3761	0.3333	
BCC		0.04167	0.8087	0.3333	
	R^2	0.9848			
	Adj R^2	0.9810			
	Std Error	0.5938			
	F	259.1617			
	Sig F	0.0000			

Factor	Name	Low	High	Exper
A	Supplier	1	3	2
B	Temp	100	200	150
C	Pressure	50	80	65

Multiple Response Prediction

	Y-hat	S-hat	99% Confidence Interval Lower Bound	99% Confidence Interval Upper Bound
Mixing Time	19.8580	0.5843	18.105	21.611

S-hat Model

Factor	Name	Coeff (Mixing Time)	P(2 Tail)	Tol	Active
Const		0.58425	0.0000		
A	Supplier	0.0000000	1.0000	0.2000	
B	Temp	0.02071	0.3047	0.2000	
C	Pressure	-0.02071	0.3047	0.2000	
AB		0.0000000	1.0000	1	
AC		0.0000000	1.0000	1	
BC		-0.01553	0.1693	1	
ABC		0.0000000	1.0000	1	
AA		-0.02071	0.1927	1	
BB		0.01036	0.5008	1	
CC		0.01036	0.5008	1	
AAB		-0.03107	0.1179	0.3333	
ABB		0.0000000	1.0000	0.3333	
AAC		0.03107	0.1179	0.3333	
ACC		0.0000000	1.0000	0.3333	
BBC		-0.01553	0.4124	0.3333	
BCC		0.01553	0.4124	0.3333	
	R^2	0.6058			
	Adj R^2	-0.0250			
	Std Error	0.0363			
	F	0.9604			
	Sig F	0.5454			

DOE PRO XL Full Factorial Design Example (cont.)

Final Regression Model

<u>DOE PRO XL > Analyze Design > Multiple Response Regression</u>

Y-hat Model — Mixing Time

Factor	Name	Coeff	P(2 Tail)	Tol	Active
Const		19.827	0.0000		
A	Supplier	2.574	0.0000	1	X
B	Temp	-1.250	0.0000	1	X
C	Pressure	-0.54630	0.0000	1	X
AA		-7.407	0.0000	1	X
BB		-0.65741	0.0000	1	X

R^2	0.9791
Adj R^2	0.9777
Std Error	0.6438
F	701.5622
Sig F	0.0000

Factor	Name	Low	High	Exper
A	Supplier	1	3	2
B	Temp	100	200	150
C	Pressure	50	80	65

Multiple Response Prediction

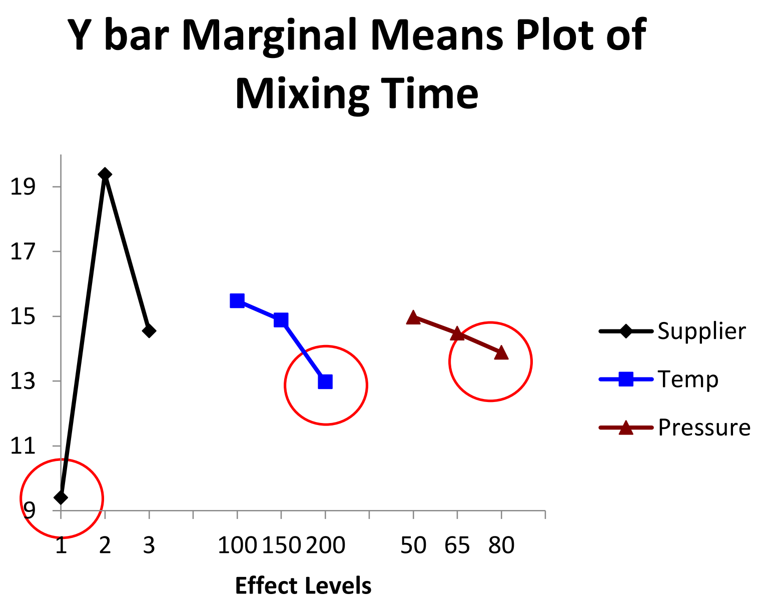
	Y-hat	S-hat	99% Confidence Interval Lower Bound	99% Confidence Interval Upper Bound
Mixing Time	19.8272	0.5843	18.074	21.580

S-hat Model — Mixing Time

Factor	Name	Coeff	P(2 Tail)	Tol	Active
Const		0.58425	0.0000		

R^2	0.0000
Adj R^2	0.0000
Std Error	0.0359
F	NA
Sig F	NA
F_{LOF}	NA
Sig F_{LOF}	NA

Source	SS	df	MS
Regression	0.0	0	NA



KISS Guidelines for Choosing and Experimental Design

KISS - Keep It Simple Statistically

Source: Steve Schmidt
Mark Kiemele
Cass Grandone

PF → **Process flow diagram**

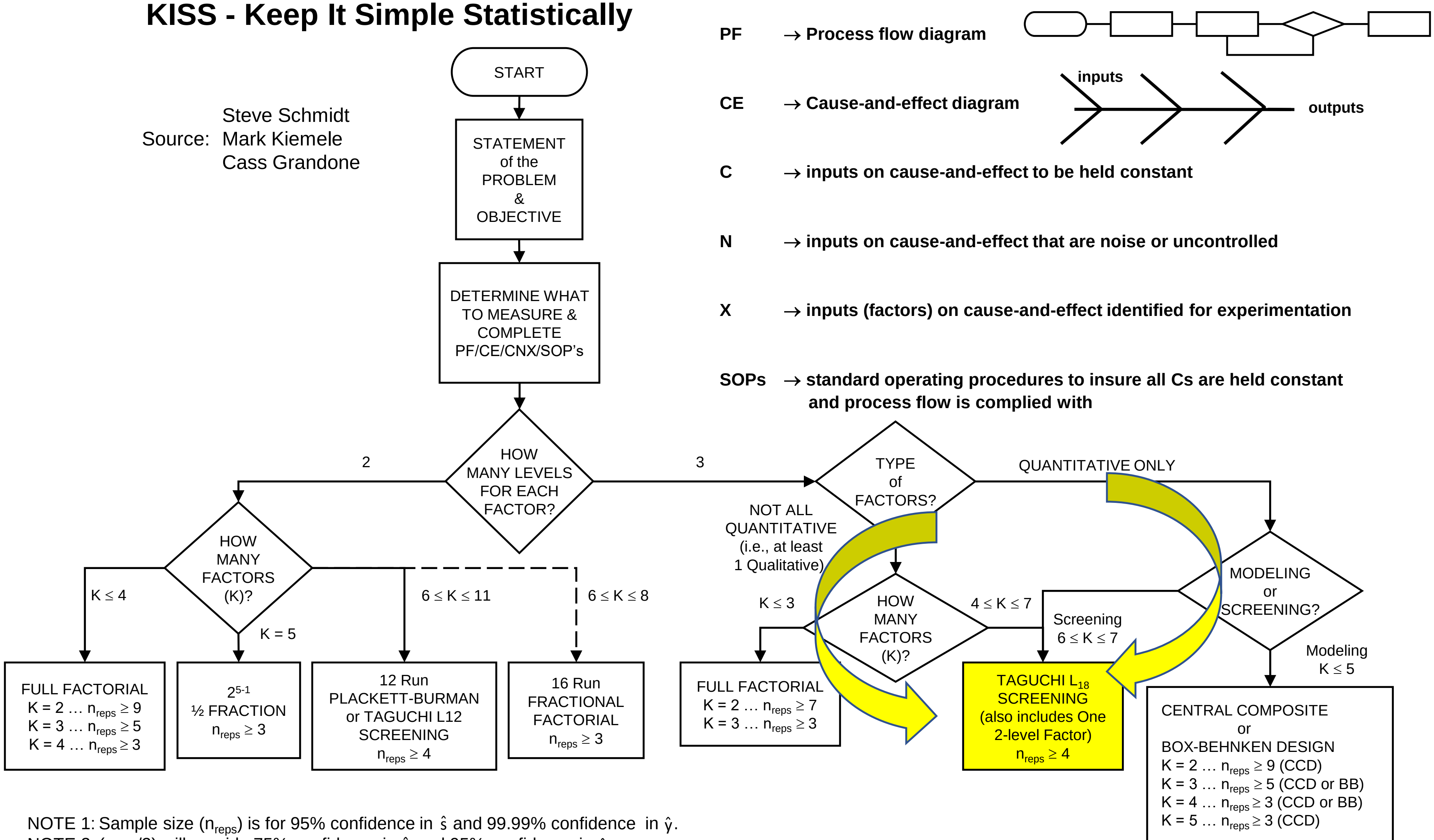
CE → **Cause-and-effect diagram**

C → **inputs on cause-and-effect to be held constant**

N → **inputs on cause-and-effect that are noise or uncontrolled**

X → **inputs (factors) on cause-and-effect identified for experimentation**

SOPs → **standard operating procedures to insure all Cs are held constant and process flow is complied with**



NOTE 1: Sample size (n_{reps}) is for 95% confidence in $\hat{s}$ and 99.99% confidence in $\hat{y}$.
NOTE 2: (n_{reps}/2) will provide 75% confidence in $\hat{s}$ and 95% confidence in $\hat{y}$.
NOTE 3: The 12 Run Plackett-Burman or L12 is very sensitive to large numbers of interactions. If this is the case, you would be better off using the 16 Run Fractional Factorial or a smaller number of variables in 2 or more full factorial experiments.
NOTE 4: For more complete 2-level design options, see next page.

Three Level Screening Design – L_{18}

OBJECTIVE: To test an orthogonal subset of the full factorial.

ADVANTAGES: Can screen many factors with just a few runs. Can estimate all main effects and all quadratics independently, as well as the AB interaction. Can mix qualitative and quantitative factors. Can handle up to seven three level factors.

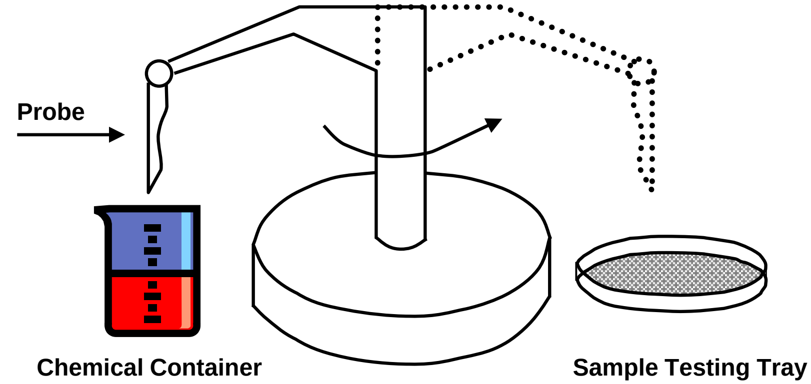
DISADVANTAGES: No direct modeling of interactions

L_{18} Design											
Run	1	2	3	4	5	6	7	8	$y_1 \ldots y_4$	$\bar{y}$	s
1	-1	-1	-1	-1	-1	-1	-1	-1			
2	-1	-1	0	0	0	0	0	0			
3	-1	-1	+1	+1	+1	+1	+1	+1			
4	-1	0	-1	-1	0	0	+1	+1			
5	-1	0	0	0	+1	+1	-1	-1			
6	-1	0	+1	+1	-1	-1	0	0			
7	-1	+1	-1	0	-1	+1	0	+1			
8	-1	+1	0	+1	0	-1	+1	-1			
9	-1	+1	+1	-1	+1	0	-1	0			
10	+1	-1	-1	+1	+1	0	0	-1			
11	+1	-1	0	-1	-1	+1	+1	0			
12	+1	-1	+1	0	0	-1	-1	+1			
13	+1	0	-1	0	+1	-1	+1	0			
14	+1	0	0	+1	-1	0	-1	+1			
15	+1	0	+1	-1	0	+1	0	-1			
16	+1	+1	-1	+1	0	+1	-1	0			
17	+1	+1	0	-1	+1	-1	0	+1			
18	+1	+1	+1	0	-1	0	+1	-1			

L_{18} Screening Design Example

Three Level Designs – Data Files

L18 Screening Example



INPUTS

Two Level → x_1 Operator, (1,2)

x_2 Probe Lots, (1,2,3)

x_3 Power Supply, (1,2,3)

x_4 Electric Motors, (1,2,3)

x_5 Chemical Lots, (1,2,3)

x_6 Pumps, (1,2,3)

Quantitative → x_7 Spec on Chem Temp, (LS,N,US)

Quantitative → x_8 Spec on Height at Dispense, (LS,N,US)

Aspiration and Dispense System Variability

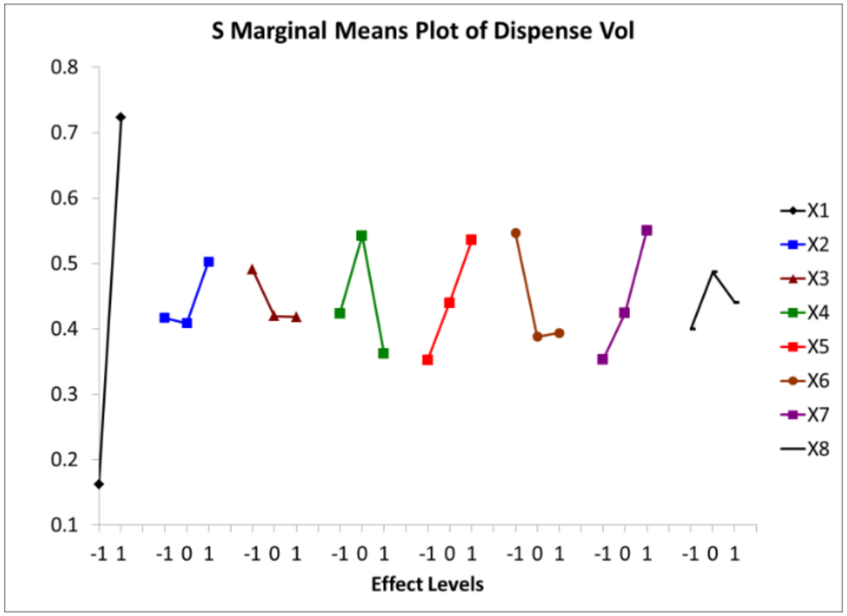
OUTPUT

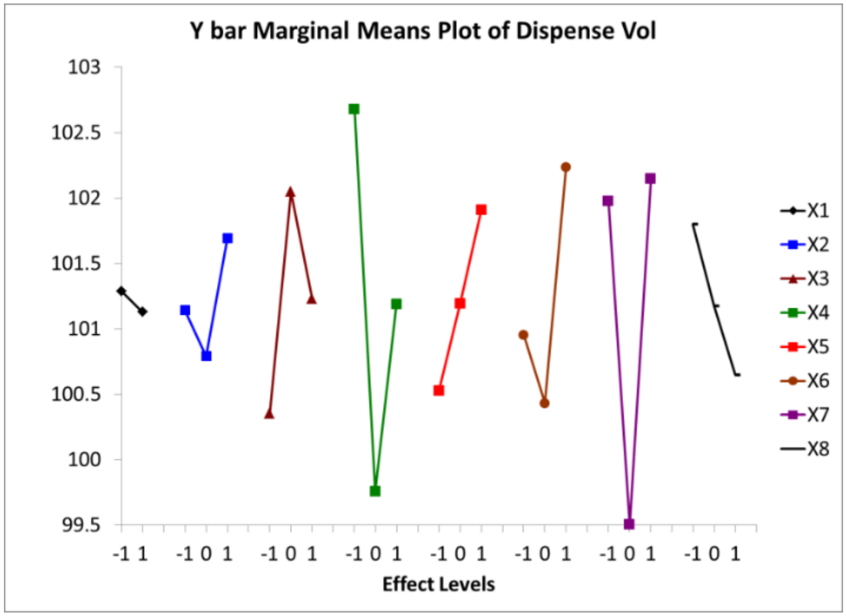
Dispense volume (y)

(Target = 100)

L_{18} Screening Design Example (cont.)

DOE PRO XL > Analyze Design > Marginal Means Plot…





L_{18} Screening Design Example (cont.)

DOE PRO XL > Analyze Design > Multiple Response Regression

Y-hat Model — Dispense Vol

Factor	Name	Coeff	P(2 Tail)	Tol	Active
Const		97.650	0.0000		
A	X1	-0.08056	0.2199	1	X
B	X2	0.27500	0.0011	1	X
C	X3	0.43958	0.0000	1	X
D	X4	-0.74583	0.0000	1	X
E	X5	0.69167	0.0000	1	X
F	X6	0.64167	0.0000	1	X
G	X7	0.08542	0.2874	1	X
H	X8	-0.57500	0.0000	1	X
AB		0.78333	0.0000	1	X
BB		0.62500	0.0000	1	X
CC		-1.256	0.0000	1	X
DD		2.175	0.0000	1	X
EE		0.02500	0.8566	1	X
FF		1.163	0.0000	1	X
GG		2.556	0.0000	1	X
HH		0.05000	0.7179	1	X
R^2		0.9559			
Adj R^2		0.9431			
Std Error		0.5508			
F		74.5448			
Sig F		0.0000			

Factor	Name	Low	High	Exper
A	X1	-1	1	0
B	X2	-1	1	0
C	X3	-1	1	0
D	X4	-1	1	0
E	X5	-1	1	0
F	X6	-1	1	0
G	X7	-1	1	0
H	X8	-1	1	0

Multiple Response Prediction

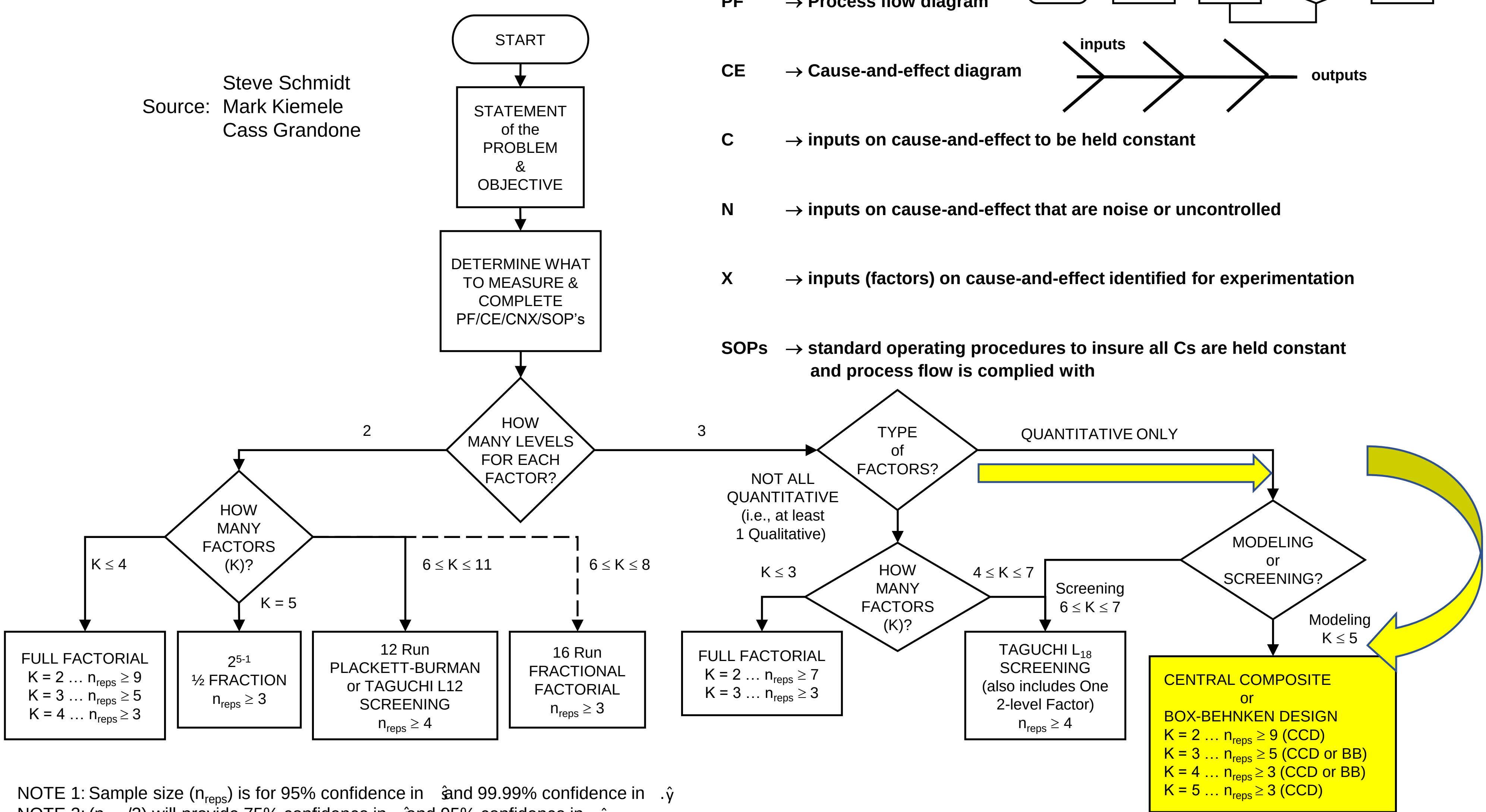
	Y-hat	S-hat	99% Confidence Interval Lower Bound	99% Confidence Interval Upper Bound
Dispense Vol	97.6500	0.4533	96.290	99.010

S-hat Model — Dispense Vol

Factor	Name	Coeff	P(2 Tail)	Tol	Active
Const		0.45331	0.1238		
A	X1	0.28003	0.0523	1	X
B	X2	0.04298	0.3701	1	X
C	X3	-0.03638	0.4203	1	X
D	X4	-0.03054	0.4752	1	X
E	X5	0.09167	0.1903	1	X
F	X6	-0.07645	0.2253	1	X
G	X7	0.09846	0.1779	1	X
H	X8	0.02019	0.6049	1	X
AB		0.09812	0.1784	1	X
BB		0.05088	0.4875	1	X
CC		0.03490	0.6055	1	X
DD		-0.14938	0.2015	1	X
EE		0.00427	0.9445	1	X
FF		0.08219	0.3418	1	X
GG		0.02786	0.6705	1	X
HH		-0.06678	0.4025	1	X
R^2		0.9953			
Adj R^2		0.9199			
Std Error		0.0978			
F		13.1950			
Sig F		0.2134			

KISS Guidelines for Choosing and Experimental Design

KISS - Keep It Simple Statistically

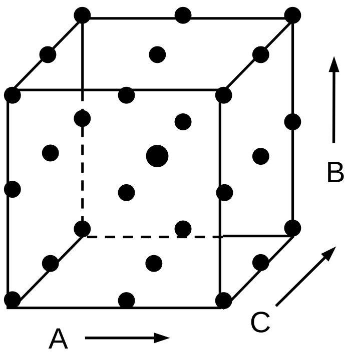


NOTE 1: Sample size (n_{reps}) is for 95% confidence in $\hat{s}$ and 99.99% confidence in $\hat{y}$.
NOTE 2: (n_{reps}/2) will provide 75% confidence in $\hat{s}$ and 95% confidence in $\hat{y}$.
NOTE 3: The 12 Run Plackett-Burman or L12 is very sensitive to large numbers of interactions. If this is the case, you would be better off using the 16 Run Fractional Factorial or a smaller number of variables in 2 or more full factorial experiments.
NOTE 4: For more complete 2-level design options, see next page.

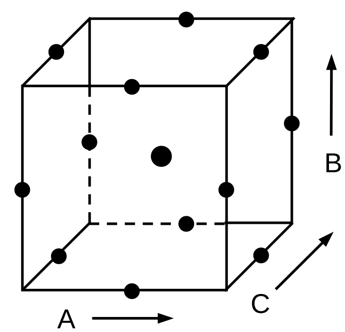
Response Surface Modeling Designs

OBJECTIVE: To test a **<u>nearly orthogonal subset of the full factorial</u>** in order to build a non-linear model for quantitative input factors (X's).

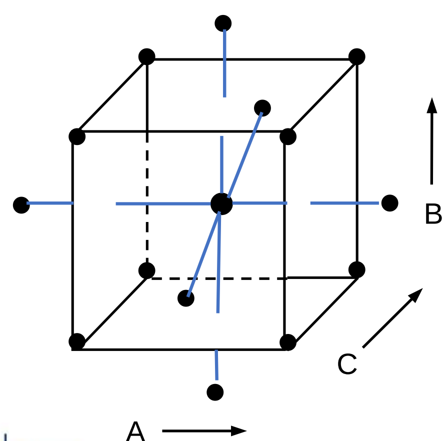
Full Factorial Design Space



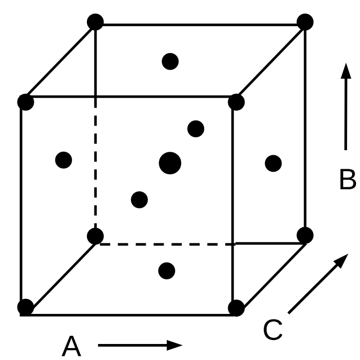
Box - Behnken Design Space



Central Composite Design (CCD) Space



Central Composite Face Design Space



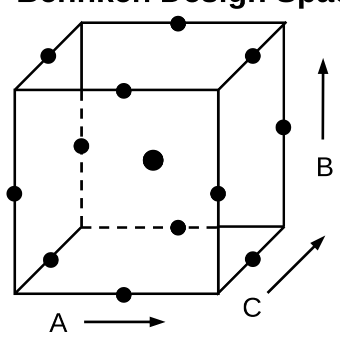
Response Surface Modeling Designs
Box - Behnken Designs

OBJECTIVE: To test a nearly orthogonal subset of the full factorial in order to build a non-linear model for quantitative input factors (X's).

ADVANTAGES: Can evaluate all main and all quadratic effects as well as all 2-way interaction effects. Much more efficient than the full factorial designs!

DISADVANTAGES: Requires quantitative factors. Not available for 2 factors and too many runs for k ≥ 5. Therefore, use only for 3 or 4 quantitative factors as shown below.

Box - Behnken Design Space



3 factors

Run	A	B	C	$y_1 \ldots y_5$		s
1	-	-	0			
2	-	+	0			
3	+	-	0			
4	+	+	0			
5	-	0	-			
6	-	0	+			
7	+	0	-			
8	+	0	+			
9	0	-	-			
10	0	-	+			
11	0	+	-			
12	0	+	+			
13	0	0	0			
14	0	0	0			
15	0	0	0			

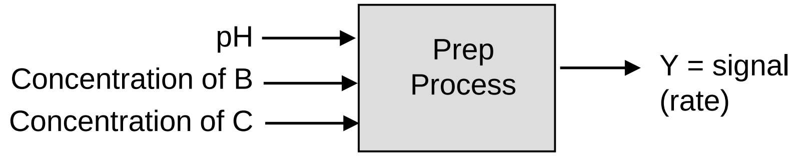
4 factors

Run	A	B	C	D	$y_1 \ldots y_4$		s
1	-	-	0	0			
2	-	+	0	0			
3	+	-	0	0			
4	+	+	0	0			
5	0	0	-	-			
6	0	0	-	+			
7	0	0	+	-			
8	0	0	+	+			
9	0	0	0	0			
10	-	0	0	-			
11	-	0	0	+			
12	+	0	0	-			
13	+	0	0	+			
14	0	-	-	0			
15	0	-	+	0			
16	0	+	-	0			
17	0	+	+	0			
18	0	0	0	0			
19	-	0	-	0			
20	-	0	+	0			
21	+	0	-	0			
22	+	0	+	0			
23	0	-	0	-			
24	0	-	0	+			
25	0	+	0	-			
26	0	+	0	+			
27	0	0	0	0			

Box Behnken Example

Box – Behnken Example

Three Level Designs – Data Files

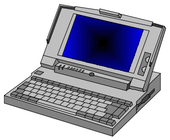


- R&D Laboratory

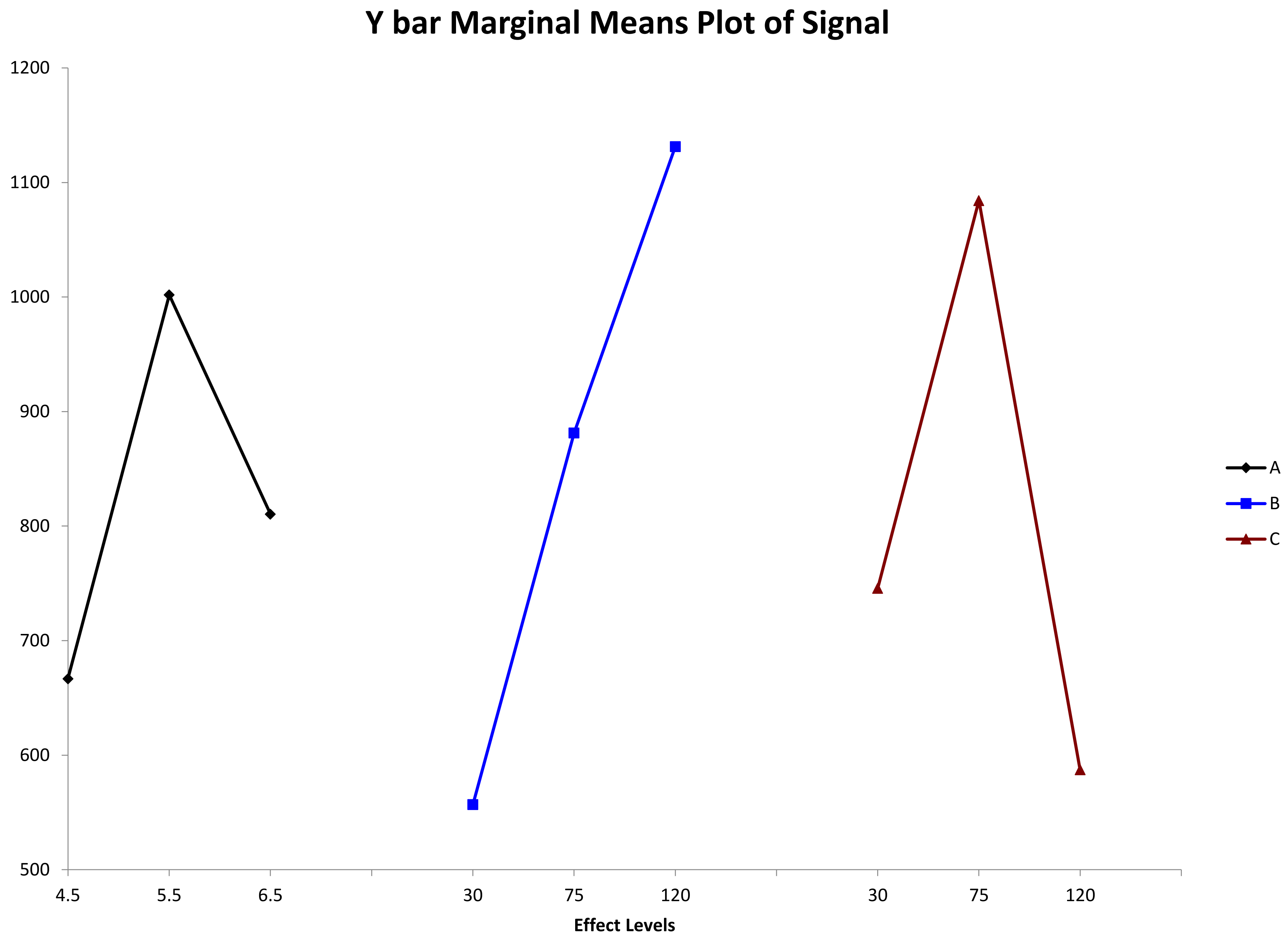
- Goal was to achieve Y = 1350
- 1 rep (although not ideal) was taken due to cost
- Most expensive factor was C
- To be competitive, highest setting for C is 45

Design Matrix with Response Data

Factor	A	B	C		
Row #	A	B	C	Y1	Y bar
1	4.5	30	75	211	211
2	4.5	120	75	1332	1332
3	6.5	30	75	959	959
4	6.5	120	75	1163	1163
5	4.5	75	30	697	697
6	4.5	75	120	427	427
7	6.5	75	30	724	724
8	6.5	75	120	396	396
9	5.5	30	30	783	783
10	5.5	30	120	275	275
11	5.5	120	30	779	779
12	5.5	120	120	1251	1251
13	5.5	75	75	1282	1282
14	5.5	75	75	1339	1339
15	5.5	75	75	1304	1304



Box – Behnken Example (cont.)



Box – Behnken Example (cont.)

First Regression Output

Factor	Name	Coeff	P(2 Tail)	Tol	Active
Const		1308.33	0.0000		
A	A	71.875	0.2085	1	X
B	B	287.13	0.0022	1	X
C	C	-79.250	0.1724	1	X
AB		-229.25	0.0226	1	X
AC		-14.500	0.8450	1	
BC		245.00	0.0177	1	X
AA		-301.54	0.0092	0.9890	X
BB		-90.542	0.2716	0.9890	
CC		-445.79	0.0017	0.9890	X

Y-hat Model — Signal

R^2	0.9570
Adj R^2	0.8795
Std Error	140.8513
F	12.3576
Sig F	0.0064

Final Regression Output

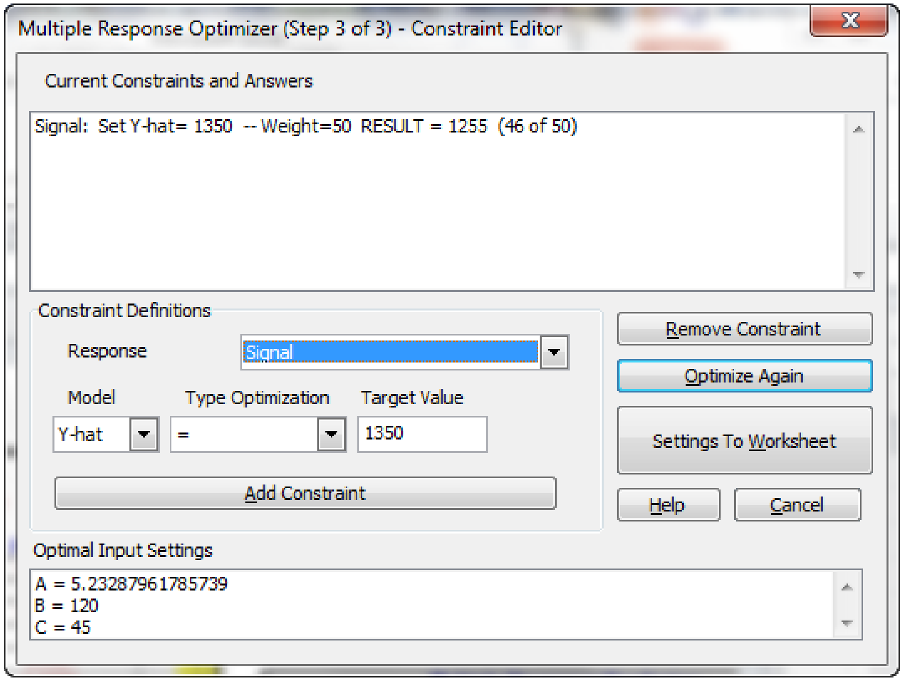
Factor	Name	Coeff	P(2 Tail)	Tol	Active
Const		1252.62	0.0000		
A	A	71.875	0.1798	1	X
B	B	287.13	0.0006	1	X
C	C	-79.250	0.1444	1	X
AB		-229.25	0.0121	1	X
BC		245.00	0.0088	1	X
AA		-294.58	0.0042	0.9949	X
CC		-438.83	0.0004	0.9949	X

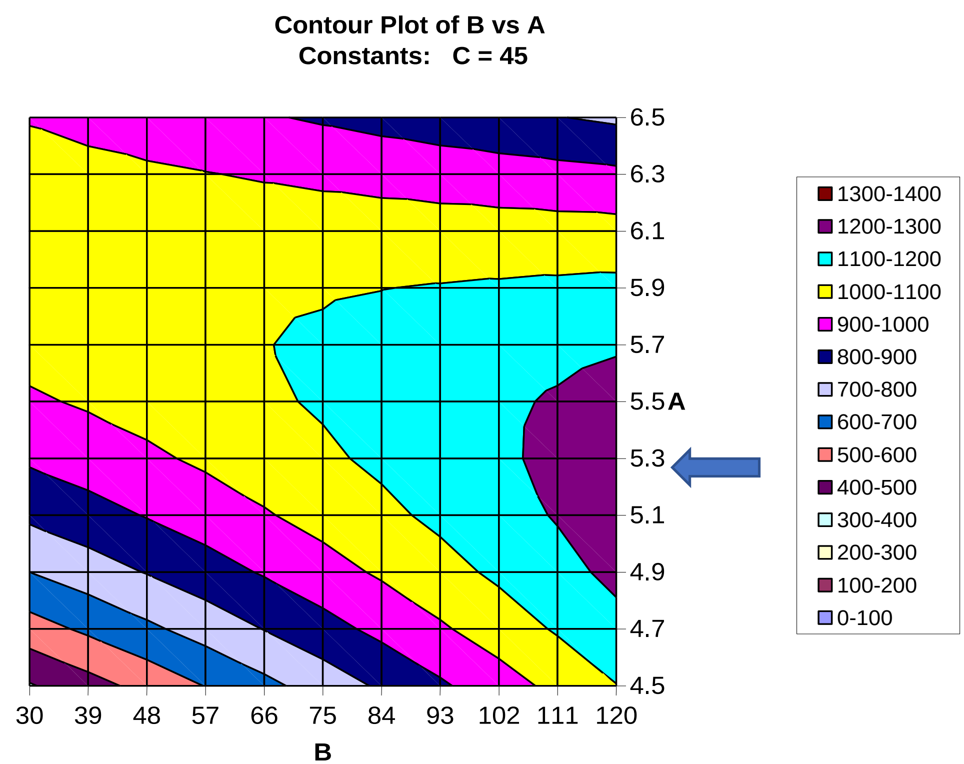
Y-hat Model — Signal

R^2	0.9435
Adj R^2	0.8870
Std Error	136.4369
F	16.6944
Sig F	0.0007

Box – Behnken Example (cont.)

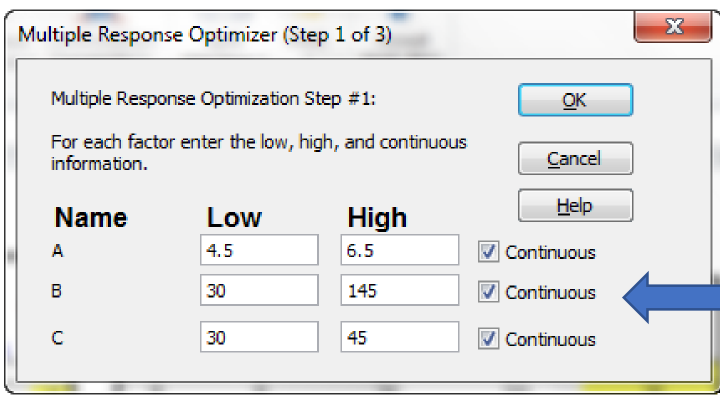
- Using Multiple Response Optimizer, is there a way to hit the target value (1350), while keeping factor C at or below 45?





Box – Behnken Example (cont.)

- The team tried several confirmation tests, within the range of the experimental settings. Keeping C at 45, they confirmed that there was no way to hit their target value. Their predictions matched well with what DOE Pro predicted.
- The team then tried extrapolating with the settings for factor B. Since their prediction model worked well within their experimental range, they decided it was worth a shot to try something outside the range as suggested by their DOE model. Note that there is no guarantee that the model will extrapolate, so confirmation is especially critical!



Factor	Name	Low	High	Exper
A	A	4.5	6.5	5.23
B	B	30	120	143
C	C	30	120	45

Multiple Response Prediction				
			99% Confidence Interval	
	Y-hat	S-hat	Lower Bound	Upper Bound
Signal	1350.1306	136.4369	940.820	1759.441

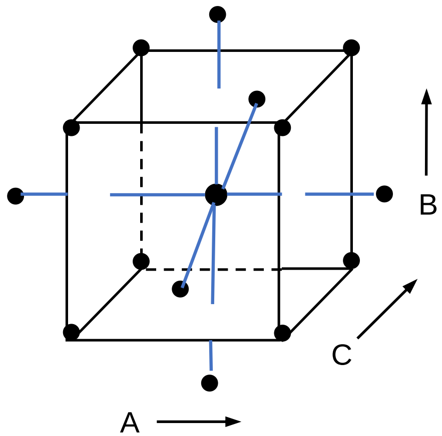
Response Surface Modeling Designs
Central Composite Designs (CCD)

OBJECTIVE: To test a nearly orthogonal subset of the full factorial in order to build a non-linear model for quantitative input factors (X's).

ADVANTAGES: Can evaluate all main and all quadratic effects as well as selected interactions (2-way and higher). Can be run sequentially: the 2-level part first and then test for linearity. If linear, no need to go further. If not, must add on axial points.

DISADVANTAGES: Primarily for quantitative factors.

Central Composite Design (CCD) Space

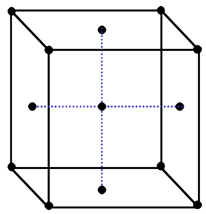


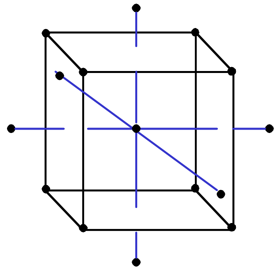
CCD for k = 3

	Run	A	B	C	
		FACTOR			
F	1	-	-	-	Factorial (2-level) portion
	2	-	-	+	
	3	-	+	-	
	4	-	+	+	
	5	+	-	-	
	6	+	-	+	
	7	+	+	-	
	8	+	+	+	
C	9	0	0	0	Centerpoint portion
	10	0	0	0	
	11	0	0	0	
A	12	$-\alpha$	0	0	Axial portion
	13	$+\alpha$	0	0	
	14	0	$-\alpha$	0	
	15	0	$+\alpha$	0	
	16	0	0	$-\alpha$	
	17	0	0	$+\alpha$	

Central Composite Designs
Suggested Values for α and # of Center Points

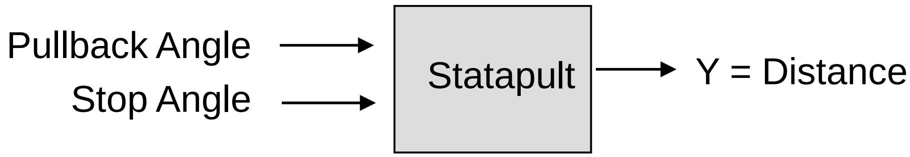
- Face-centered Design ($\alpha = 1$)
  - Hard limits (restrictions) on factor settings
  - Cannot take factor settings beyond ±1 (coded values)
  - Predictions made within the "cube"
  - Recommended number of center points = 2
  - Orthogonality is worse with more than 2 center points





- Spherical Design ($\alpha = \sqrt{k}$)
- Rotatable Design ($\alpha = (n_F)^{1/4}$)
  - k is the number of factors; n_F is the number of runs in the factorial part of the design
  - No hard limits (constraints) on factor settings
  - Able to go beyond ±1 coded settings
  - Predictions slightly beyond the "cube" (in case the optimum lies just outside)
  - Orthogonality improves with more center points; 3-6 is recommended

Response Surface Modeling Designs
Central Composite Designs (CCD) Example



2 Factor - CCD Template from DOE PRO XL

	Run	**pull back**	**stop angle**		**Y1**	**Y2**	**Y3**	**Y4**
Factorial	***1***	- 160	- 2					
	2	- 160	+ 4					
	3	+ 180	- 2					
	4	+ 180	+ 4					
Center	***5***	0 170	0 3					
	6	0 170	0 3					
Axial	***7***	- 160	0 3					
	8	+ 180	0 3					
	9	0 170	- 2					
	10	0 170	+ 4					

Response Surface Modeling Designs
Central Composite Designs (CCD) Example (cont.)

CCD Example
Factorial
Portion

Three Level Designs – Data Files

- Collect data and complete the factorial portion of the CCD.

Factor	A	B		Distance						
Row #	pullback angle	stop angle		Y1	Y2	Y3	Y4		Y bar	S
1	160	2		27.5	27.5	27.5	27		27.375	0.25
2	160	4		47	47	48	48		47.5	0.57735
3	180	2		64.5	64	63.5	62		63.5	1.080123
4	180	4		77	74	75.5	75		75.375	1.25
5	170	3							#DIV/0!	#DIV/0!
6	170	3							#DIV/0!	#DIV/0!
7	160	3							#DIV/0!	#DIV/0!
8	180	3							#DIV/0!	#DIV/0!
9	170	2							#DIV/0!	#DIV/0!
10	170	4							#DIV/0!	#DIV/0!

- Select Analyze Design from DOE PRO XL with only this data and duplicate the regression output on the next page!

Response Surface Modeling Designs
Central Composite Designs (CCD) Example (cont.)

Y-hat Model — Distance

Factor	Name	Coeff	P(2 Tail)	Tol	Active
Const		53.438	0.0000		
A	pullback angle	16.000	0.0000	1	X
B	stop angle	8.000	0.0000	1	X
AB		-2.063	0.0000	1	X
AA					
BB					

R^2	0.9982
Adj R^2	0.9977
Std Error	0.8839
F	2213.5733
Sig F	0.0000

Factor	Name	Low	High	Exper
A	pullback angle	160	180	170
B	stop angle	2	4	3

Multiple Response Prediction

	Y-hat	S-hat	99% Confidence Interval Lower Bound	99% Confidence Interval Upper Bound
Distance	53.4375	0.7894	51.069	55.806

S-hat Model — Distance

Factor	Name	Coeff	P(2 Tail)	Tol	Active
Const		0.78937	NA		
A	pullback angle	0.37569	NA	1	X
B	stop angle	0.12431	NA	1	
AB		-0.03937	NA	1	
AA					
BB					

R^2	1.0000
Adj R^2	NA
Std Error	NA
F	NA
Sig F	NA

- Fantastic model with good results!

- Row five data at the centerpoints is: 57, 57.5, 57, 57!
- Did not confirm! We need to collect the remaining rows of the CCD design (the Center and Axial portions of the design)!

Response Surface Modeling Designs
Central Composite Designs (CCD) Example (cont.)

- Collect the remaining data and complete the CCD design.

CCD Example
Full Data Set

Three Level Designs – Data Files

Factor	A	B		Distance						
Row #	pullback angle	stop angle		Y1	Y2	Y3	Y4		Y bar	S
1	160	2		27.5	27.5	27.5	27		27.375	0.25
2	160	4		47	47	48	48		47.5	0.57735
3	180	2		64.5	64	63.5	62		63.5	1.080123
4	180	4		77	74	75.5	75		75.375	1.25
5	170	3		57	57.5	57	57		57.125	0.25
6	170	3		57.5	57	56.5	57		57	0.408248
7	160	3		48	47	47	47		47.25	0.5
8	180	3		73.5	75	73	74.5		74	0.912871
9	170	2		43	42	42	42		42.25	0.5
10	170	4		58	58	61	58.5		58.875	1.436141

- Select Analyze Design from DOE PRO XL with the complete data set and duplicate the regression output on the next page!

Response Surface Modeling Designs
Central Composite Designs (CCD) Example (cont.)

Y-hat Model

Factor	Name	Coeff (Distance)	P(2 Tail)	Tol	Active
Const		57.259	0.0000		
A	pullback angle	15.125	0.0000	1	X
B	stop angle	8.104	0.0000	1	X
AB		-2.063	0.0000	1	X
AA		3.170	0.0000	0.9722	X
BB		-6.893	0.0000	0.9722	X

R^2	0.9924
Adj R^2	0.9912
Std Error	1.3132
F	883.4346
Sig F	0.0000

Factor	Name	Low	High	Exper
A	pullback angle	160	180	170
B	stop angle	2	4	3

Multiple Response Prediction

	Y-hat	S-hat	99% Confidence Interval Lower Bound	99% Confidence Interval Upper Bound
Distance	57.2589	0.4880	55.795	58.723

S-hat Model

Factor	Name	Coeff (Distance)	P(2 Tail)	Tol	Active
Const		0.48799	0.0581		
A	pullback angle	0.31927	0.0653	1	X
B	stop angle	0.23889	0.1323	1	
AB		-0.03937	0.8122	1	
AA		0.05959	0.7838	0.9722	
BB		0.32122	0.1889	0.9722	

R^2	0.7619
Adj R^2	0.4643
Std Error	0.3102
F	2.5598
Sig F	0.1917

- One final regression to clean up the insignificant terms!

Response Surface Modeling Designs
Central Composite Designs (CCD) Example (cont.)

Y-hat Model — Distance

Factor	Name	Coeff	P(2 Tail)	Tol	Active
Const		57.259	0.0000		
A	pullback angle	15.125	0.0000	1	X
B	stop angle	8.104	0.0000	1	X
AB		-2.063	0.0000	1	X
AA		3.170	0.0000	0.9722	X
BB		-6.893	0.0000	0.9722	X

R^2	0.9924
Adj R^2	0.9912
Std Error	1.3132
F	883.4346
Sig F	0.0000

Factor	Name	Low	High	Exper
A	pullback angle	160	180	170
B	stop angle	2	4	3

Multiple Response Prediction

	Y-hat	S-hat	99% Confidence Interval Lower Bound	99% Confidence Interval Upper Bound
Distance	57.2589	0.7165	55.110	59.408

S-hat Model — Distance

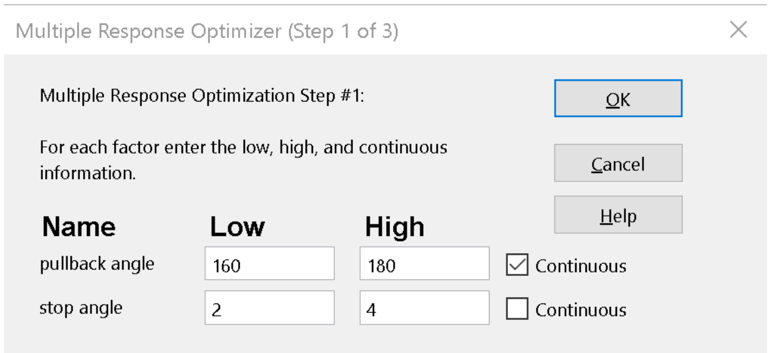
Factor	Name	Coeff	P(2 Tail)	Tol	Active
Const		0.71647	0.0002		
A	pullback angle	0.31927	0.0584	1	X

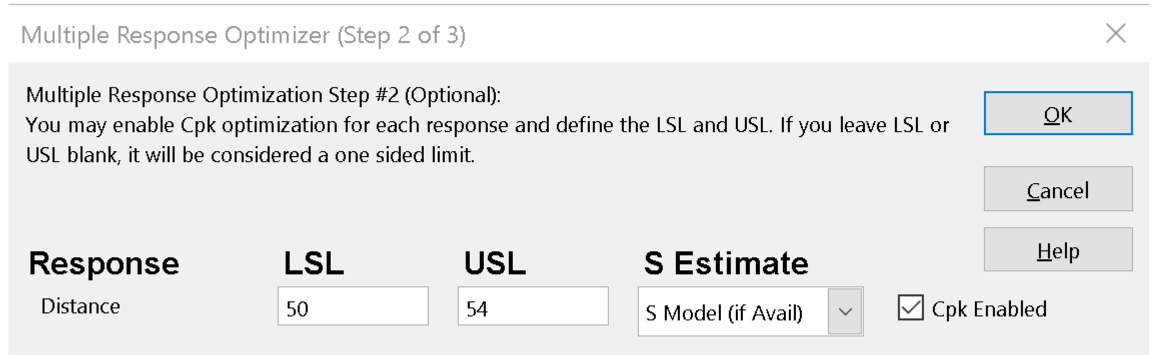
R^2	0.3783
Adj R^2	0.3006
Std Error	0.3544
F	4.8684
Sig F	0.0584
F_{LOF}	0.2207
Sig F_{LOF}	0.6528

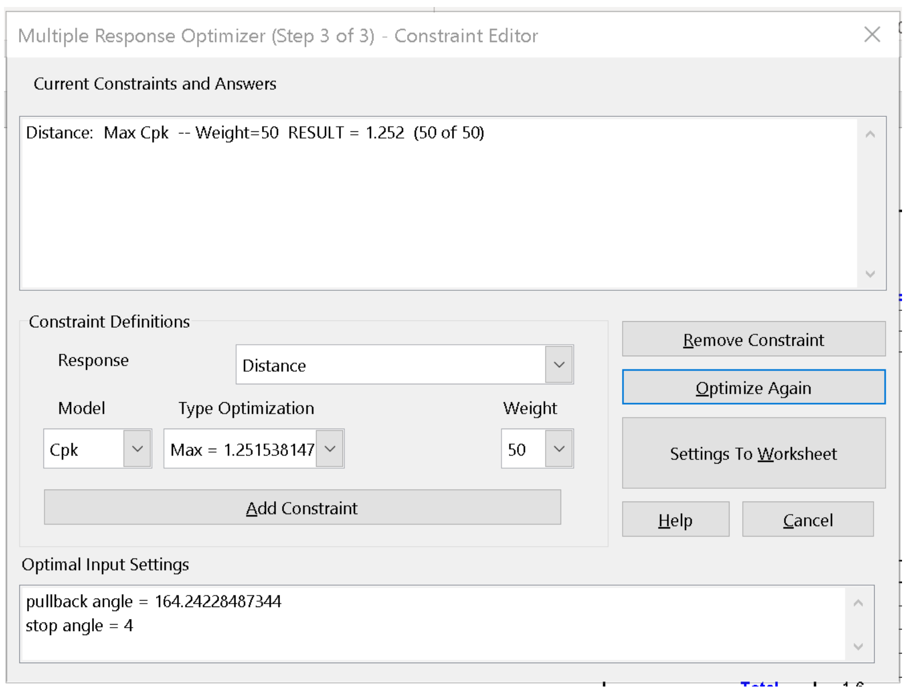
Source	SS	df	MS

- We now have a non-linear model for Y-hat and a potential linear model for S-hat! If the customer gives us a target, we now will be able to determine input settings to hit the target consistently! The target is 52 with a lower specification of 50 and an upper specification of 54. What are the input settings to hit this target consistently?

Response Surface Modeling Designs
Central Composite Designs (CCD) Example (cont.)



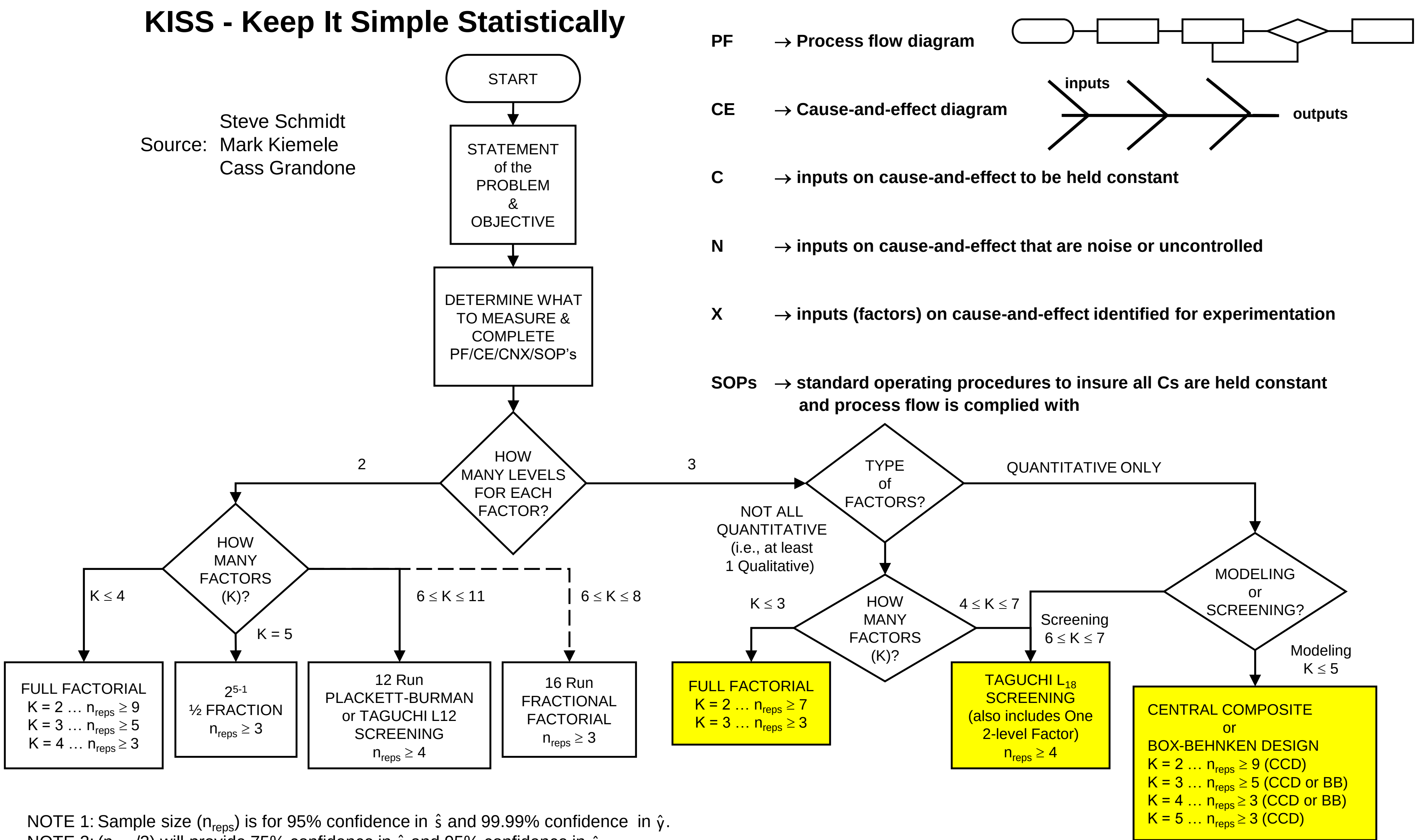




Factor	Name	Low	High	Exper
A	pullback angle	160	180	164.2422849
B	stop angle	2	4	4

Multiple Response Prediction				
			99% Confidence Interval	
	Y-hat	S-hat	Lower Bound	Upper Bound
Distance	52.0000	0.5326	50.402	53.598

KISS Guidelines for Choosing an Experimental Design



NOTE 1: Sample size (n_{reps}) is for 95% confidence in $\hat{s}$ and 99.99% confidence in $\hat{y}$.
NOTE 2: ($n_{reps}/2$) will provide 75% confidence in $\hat{s}$ and 95% confidence in $\hat{y}$.
NOTE 3: The 12 Run Plackett-Burman or L12 is very sensitive to large numbers of interactions. If this is the case, you would be better off using the 16 Run Fractional Factorial or a smaller number of variables in 2 or more full factorial experiments.
NOTE 4: For more complete 2-level design options, see next page.

Key Takeaways

- As a review techniques, stop the video and summarize the key learnings from this session. When you are finished, continue to the next page.

Key Takeaways

- Full Factorial designs are great for mixed factors (at least one qualitative input factor) when K (number of factors) is 3 or less!
- The L_{18} is a great screening design for mixed factors and larger K!
- The marginal means plots are used extensively with the full factorial and L_{18} design when some of the factors are qualitative.
- Models do not make sense for qualitative input factors.
- The Box-Behnken and CCD designs are more efficient than the full factorial designs. For quantitative input factors only (typically K = 5 or less, these designs are perfect for model building with considerable less resources!
- The CCD design can be run sequentially. Keep it simple statistically (KISS) – stay linear (two-level input factors) until shown otherwise. The center section is the confirmation of the linear model. If it confirms, congrats, you are complete (resources savings)! If it does not confirm, collect the center and axial pieces. Combine with the factorial piece and build the non-linear model.
- DOE PRO XL is nice software that has good graphical and optimization tools!

Supplemental Material

- Suggested Reading:
  - ***Lean Six Sigma: A Tools Guide*** by Adams, Kiemele, Pollock and Quan (pp. 139 - 146)
  - ***Basic Statistics – Tools for Continuous Improvement*** by Kiemele, Schmidt and Berdine, 4th edition (Chapter 8)
  - ***Design for Six Sigma: The Tool Guide for Practitioners*** by Reagan and Kiemele (section 7.9)
  - ***Understanding Industrial Designed Experiments*** by Schmidt and Launsby (chapter 2, 3, and 5)
  - Air Academy's app: ***Six Sigma Quick Tools***

GET IT ON Google Play

Download on the App Store

- SPC XL™ software training tutorials:
  - https://airacad.com/our-insights/training-videos/spc-xl/
- The data files for this session can be downloaded from the site where you are accessing this course.

Additional Practice / Review Questions

- For each of the following scenarios, identify the design (and corresponding sample size) you would recommend:

  - You have just completed a screening experiment and determined there are three critical factors in a process under study. All of the factors are quantitative and you suspect nonlinearities and several significant two factor interactions. You'd like to be able to build a nonlinear model.

  - You are studying a fairly new injection molding process. You and your team have identified a total of 5 potentially important factors to study. The factors are: a) material vendor; b) holding time; c) holding pressure; d) gate size; and e) mold temperature. You'd like to determine which of these five factors has the most significant effect on percent shrinkage for further investigation.

  - You want to study three factors in a process to determine a good combination for giving optimum performance. One of the factors (brand) is qualitative, and the other two are quantitative (time and temperature). You suspect nonlinearities and interactions amongst the factors.

  - You want to study three input factors in a chemical process to optimize the input factors for best performance. All of the input factors are quantitative. You are not sure of nonlinearities for the ranges of the input factors selected. You do suspect interactions amongst the input factors.

We can help…
Connect With Us

Remote Project Coaching

There are times when help outside your organization is needed. When that time comes, benefit from a partner that is experienced, tested, and trusted.

Expert coaching is one of the Top Five Best Practices for generating step change in project execution, as well as enhanced return on investment. We can work remotely with your organization to provide coaching support.

Air Academy Associates
Phone: (719) 531-0777
Email: aaa@airacad.com
https://airacad.com/
https://sixsigmaproductsgroup.com/

Download on the App Store

There's an app for that!
Six Sigma Quick Tools

